Lecture 25: Learning 4

Victor R. Lesser

CMPSCI 683

Fall 2010

Final Exam Information

- Final EXAM on Th 12/16 at 4:00pm in Lederle Grad Res Ctr Rm A301
  - 2 Hours but obviously you can leave early!
- Open Book but no access to Internet
- Material from Lectures 12 -25
  - Lecture 14 will not be covered on exam
  - More operational than conceptual in that I will require you to carry out steps of an algorithm or inference process

Today's Lecture

- Reinforcement Learning

Problem with Supervised Learning

- Supervised learning is sometimes unrealistic: where will correct answers come from?
  - New directions emerging in the use of redundant information as a way of getting around the lack of extensive training data

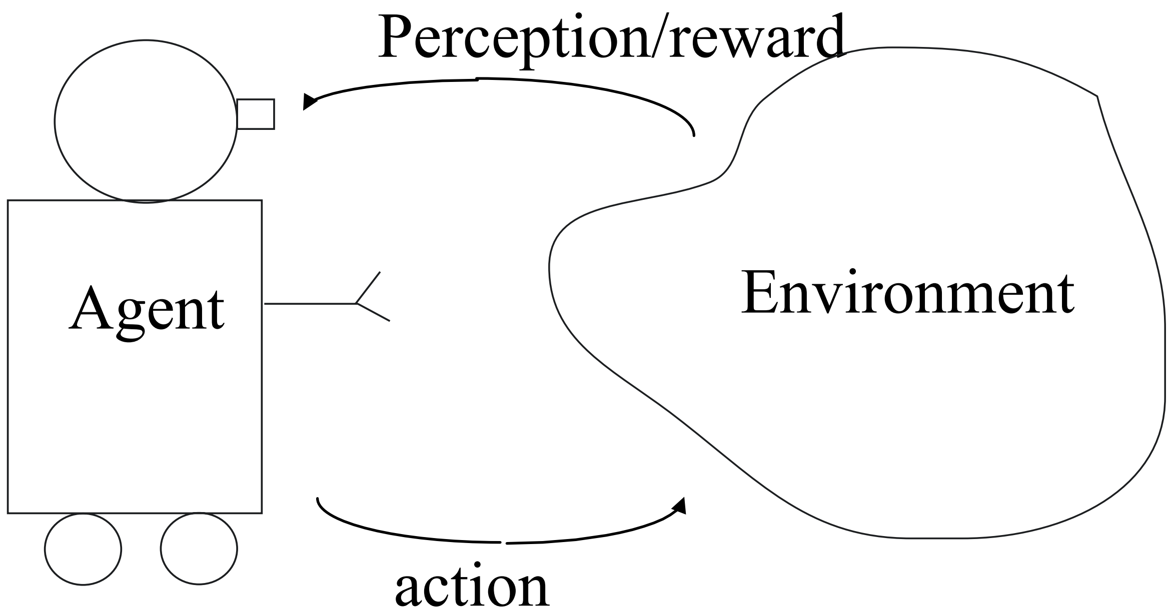
- *In many cases, the agent will only receive a single reward, after a long sequence of actions/decisions.*

- *Environments change, and so the agent must adjust its action choices.*
  - *On-line issue*

Reinforcement Learning

- Using feedback/rewards to learn a successful agent function.
- Rewards may be provided following each action, or only when the agent reaches a terminal state.
- Rewards can be components of the actual utility function or they can be hints ("nice move", "bad dog", etc.).

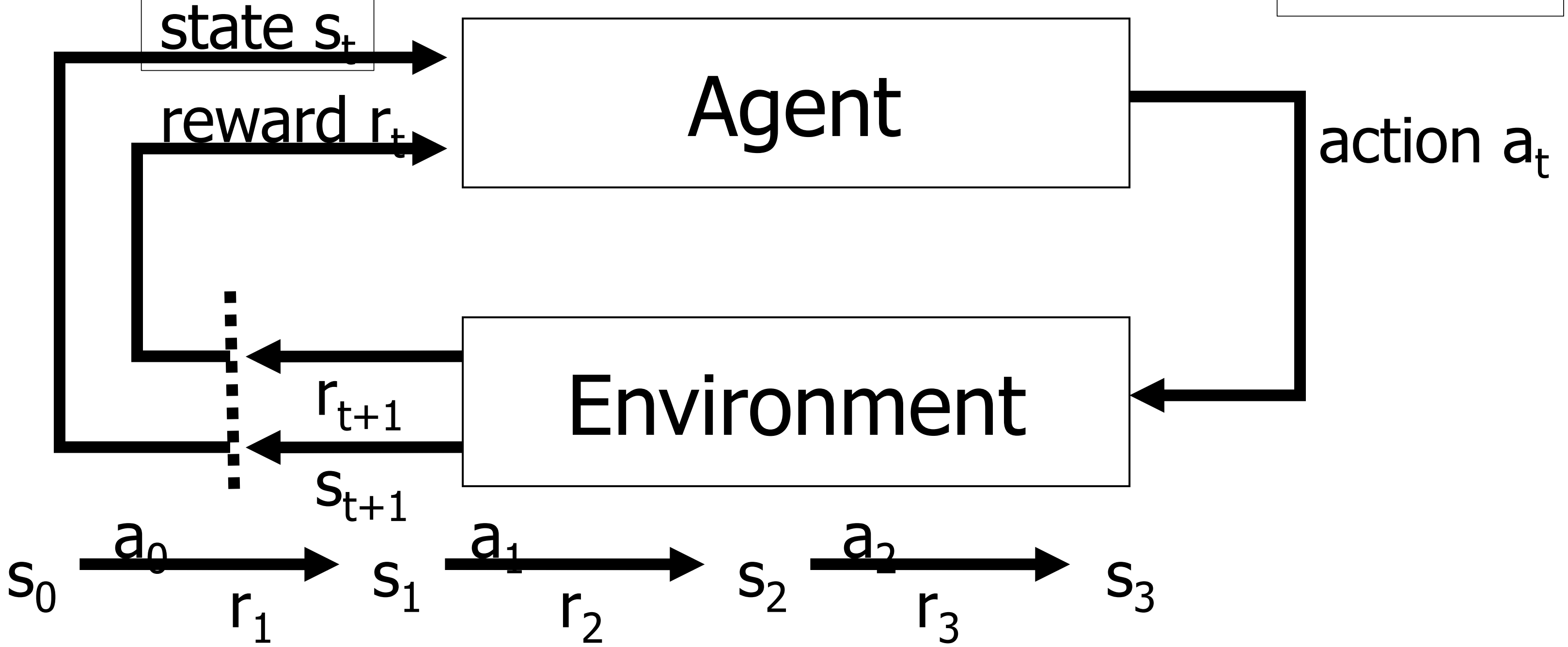
Reinforcement Learning



Utility(reward) depends on a sequence of decisions

How to learn best action (maximize expected reward) to take at each state of Agent

Reinforcement Learning Problem



Agent and environment interact at discrete time steps: $t = 0, 1, 2, K$

- Agent observes state at step t: $s_t \in S$
- produces action at step t: $a_t \in A(s_t)$
- gets resulting reward: $r_{t+1} \in \Re$
- and resulting next state: s_{t+1}

RL and Markov Decision Processes

- S - finite set of domain states
- A - finite set of actions
- $P(s'|s,a)$ - state transition function
- $r(s,a)$ - reward function
- S_0 - initial state
- The Markov assumption:

$$P(s_t \mid s_{t-1}, s_{t-2}, \ldots, s_1, a) = P(s_t \mid s_{t-1}, a)$$

RL Learning Task

Execute actions in the environment, observe results and

- Learn a policy $\pi(s) : S \rightarrow A$ from states $s_t \in S$ to actions $a_t \in A$ that maximizes the expected reward : $E[r_t + \gamma\, r_{t+1} + \gamma^2 r_{t+2} + \ldots]$ from any starting state s_t
- $0 < \gamma < 1$ is the discount factor for future rewards
- Target function is $\pi(s) : S \rightarrow A$
- But there are no direct training examples of the form <s,a>, i.e., what action is the right one to take in state s
- *Training examples are of the form <<s,a,s'>,r>*

Key Features of Reinforcement Learning

- Learner is not told which actions to take
  - Learning about, from, and while interacting with an external environment
- Trial-and-Error search
- Possibility of delayed reward
  - Sacrifice short-term gains for greater long-term gains
- *The need to explore and exploit*
  - On-line Integrating performance and learning
- Considers the whole problem of a goal-directed agent interacting with an uncertain environment

Reinforcement Learning: Two Approaches

- Learning Model of Markov Decision Process
  - Learn model of operators transitions and their rewards
  - Compute optimal policy (value/policy iteration) based on model
- Learning Optimal Policy Directly
  - You don’t necessarily need to explicit learn MDP model in order to compute optimal policy

Two basic designs

- Utility-based agent learns a *Utility function* on states (or histories) which can be used in order to select actions
  - Must have a model of the environment
  - Know the result of the action (what state the action leads to)
- Q-learning agent learns an *Action-value function* for each state (also called Q-learning; does not require a model of the environment)
  - Does not need a model of the environment, only compare its available choices
  - Can not look ahead because do not know where their actions lead.

Utility function and action-value function

- Utility function denotes the reward for starting in state s and following policy π.

$$U^{\pi}(s) = r_t + \gamma\, r_{t+1} + \gamma^2\, r_{t+2} + \ldots = \Sigma_{i=0}\, \gamma^i\, r_{t+i}$$

- Action value function denotes the reward for starting in state s, taking action a and following policy π afterwards.

$$Q^{\pi}(s,a) = r(s,a) + \gamma\, r_{t+1} + \gamma^2\, r_{t+2} + \ldots = r(s,a) + \gamma U^{\pi}(\pi(s,a))$$

Optimal Value Functions and Policies

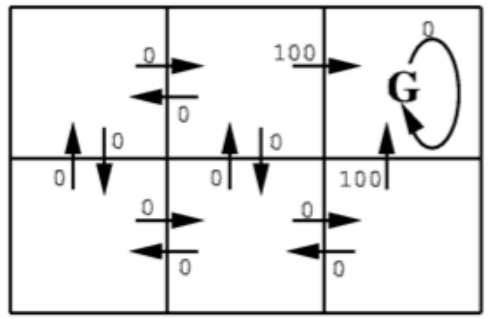
There exist optimal value functions:

$$V^*(s) = \max_\pi V^\pi(s) \qquad Q^*(s,a) = \max_\pi Q^\pi(s,a)$$

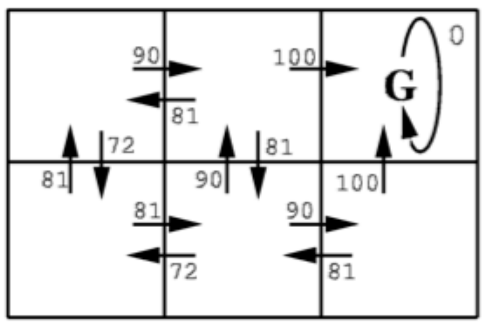
And corresponding optimal policies:

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

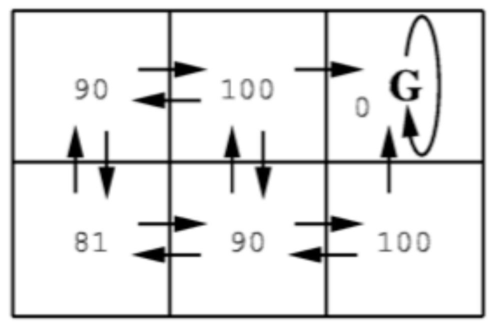
π^* is the greedy policy with respect to Q^*



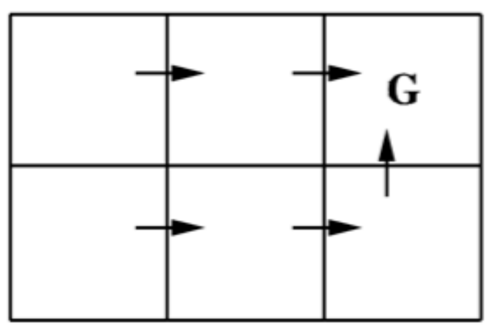
$r(s, a)$ (immediate reward) values



$Q(s, a)$ values



$V^*(s)$ values



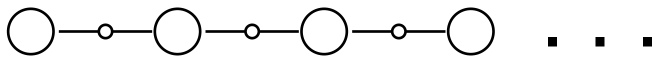
One optimal policy

Passive versus Active learning

- A *passive learner* simply watches the world going by, and tries to learn the utility of being in various states.

- An *active learner* must also act using the learned information, and can use its problem generator to suggest explorations of unknown portions of the environment.

What Many RL Algorithms Do



Experience

Build Predictions

Value Function

$V \rightarrow V^*$

$Q \rightarrow Q^*$

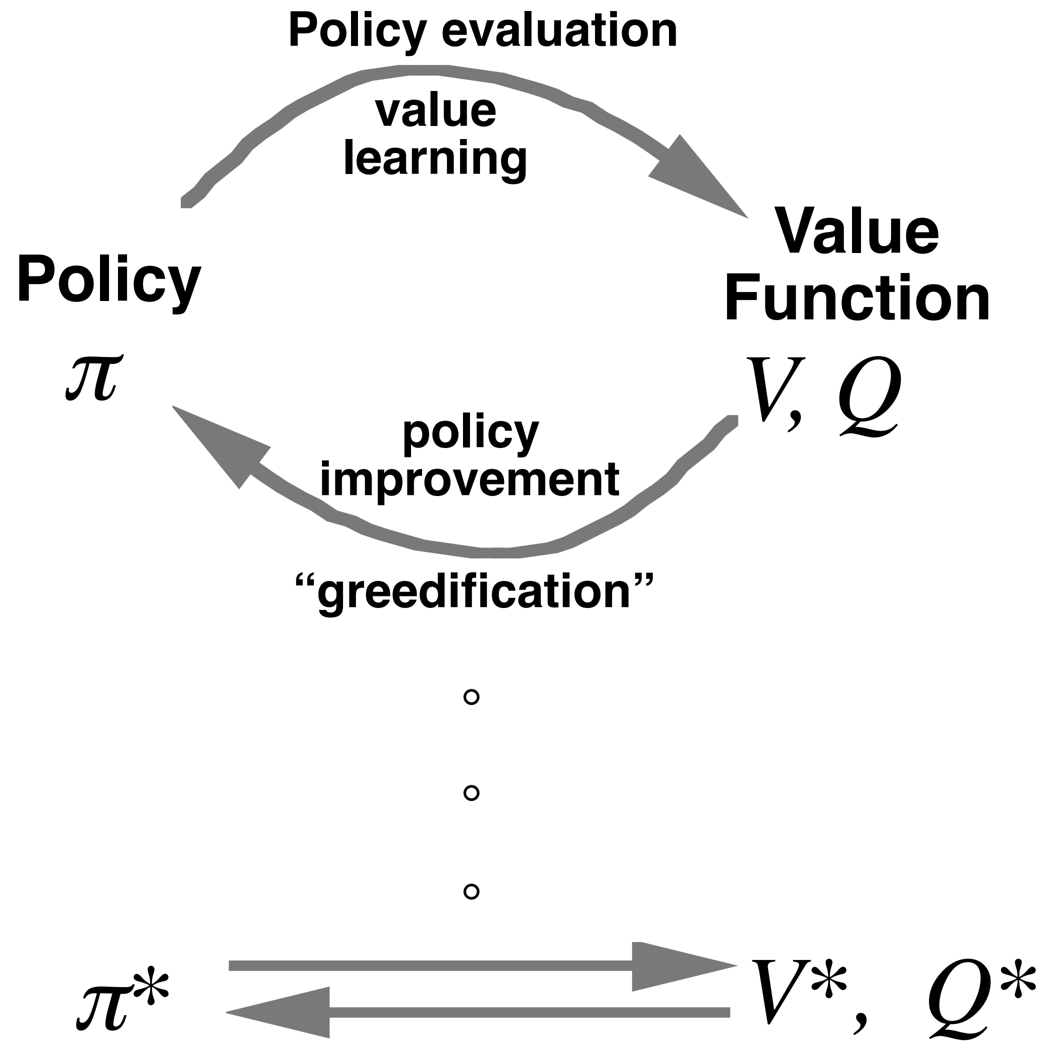
Select Actions

Policy

$\pi \rightarrow \pi^*$

- **Continual, online**
- **Simultaneous acting and learning**

RL Interaction of Policy and Value



Passive Learning in Known Environment

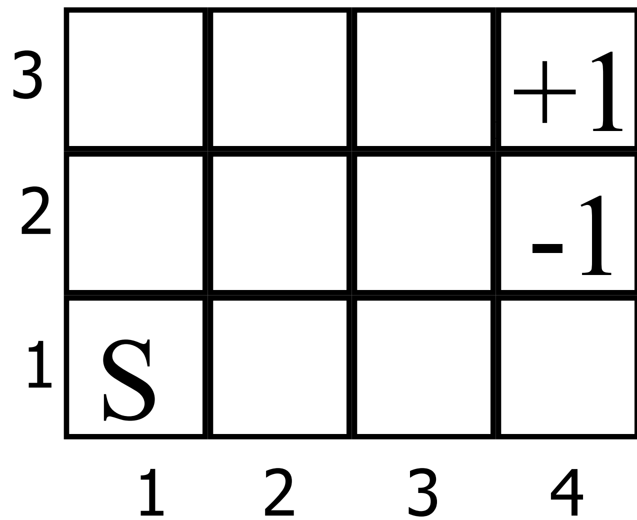
Given:

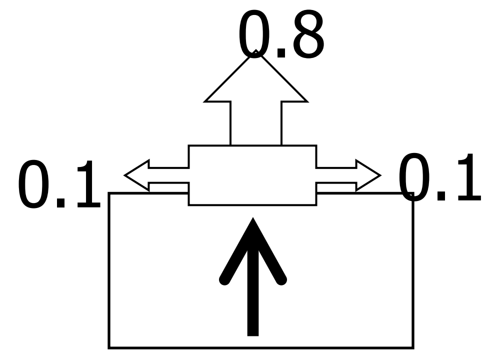
- A Markov model of the environment.
  - $P(s,s',a)$ – probability of transition from s to s' given a
  - $R(s,s',a)$ – expected reward on transition s to s' given a
- States, with probabilistic actions.
- Terminal states have rewards/utilities.

Problem:

- Learn expected utility of each state *V(s) or U(s)*.

Example





- Non-deterministic actions (transition model unknown to agent)
- Every state besides terminal states has reward -0.04
- Percepts tell you: [State, Reward, Terminal?]
- 3 Sequences of (state,action,reward)

$(1,1)_{-.04} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (2,3)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (4,3)_{+1}$

$(1,1)_{-.04} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (2,3)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (3,2)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (4,3)_{+1}$

$(1,1)_{-.04} \rightarrow (2,1)_{-0.4} \rightarrow (3,1)_{-0.4} \rightarrow (3,2)_{-0.4} \rightarrow (4,2)_{-1}$

Learning Utility Functions

- A training sequence is an instance of world transitions from an initial state to a terminal state.
- The additive utility assumption: utility of a sequence is the sum of the rewards over the states of the sequence.
- *Under this assumption, the utility of a state is the expected reward-to-go of that state*.

Direct Utility Estimation*

- Developed in the late 1950's in the area of adaptive control theory.
- Just keep a running average of rewards for each state.
- *For each training sequence, compute the reward-to-go for each state in the sequence and update the utilities.*

$(1,1)_{-.04} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (2,3)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (4,3)_{+1}$

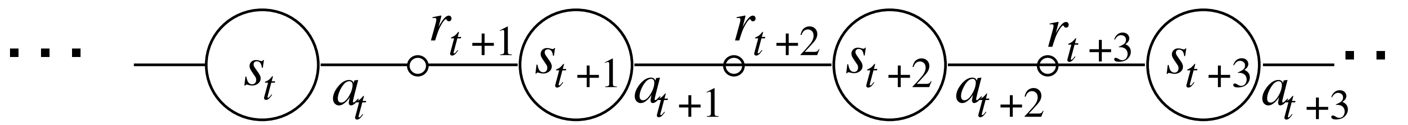
0.72 0.76 0.80 0.84 0.88 0.92 0.96 1.0

$U(1,1) = 0.72$; $U(2,3) = 0.92$; $U(3,3) = 0.96$; $U(4,3) = 1.0$;

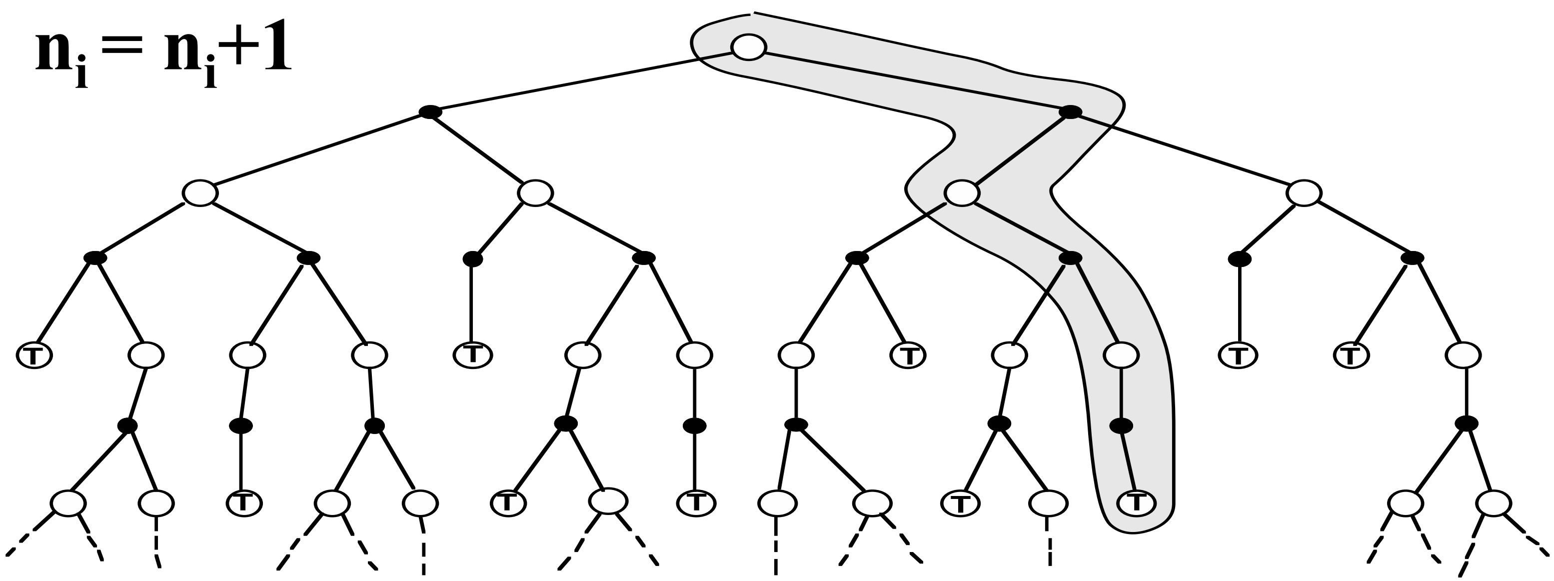
$U(1,2) = (0.76 + 0.84)/2 = 0.80$

$U(1,3) = (0.80 + 0.88)/2 = 0.84$

Direct Utility Estimation, cont



- i = $\boldsymbol{s_t}$
- ***Reward-to-go (i) = sum of*** $\boldsymbol{r_{t+1} + r_{t+2} + \ldots\ r_{terminal}}$
- **$\mathbf{U(i)_{ni+1} = (U(i)_{ni}}$ + reward-to-go (i))/ $\mathbf{(n_i+1)}$**
- **$\mathbf{n_i = n_i+1}$**



Problems with Direct Utility Estimation

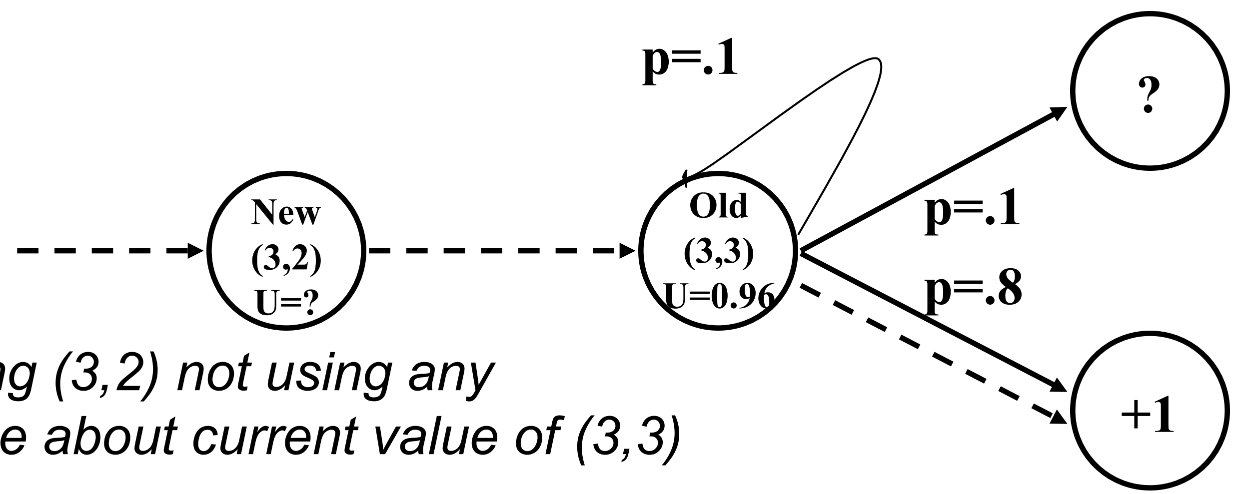
Converges very slowly because it ignores the relationship between neighboring states:

$(1,1)_{-.04} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (2,3)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (4,3)_{+1}$

0.72 0.76 0.80 0.84 0.88 0.92 0.96 1.0

$U(1,1) = 0.72;\quad U(2,3) = 0.92;\quad U(3,3) = 0.96;\quad U(4,3) = 1.0;$

$(1,1)_{-.04} \rightarrow (1,2)_{-0.4} \rightarrow (1,3)_{-0.4} \rightarrow (2,3)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (3,2)_{-0.4} \rightarrow (3,3)_{-0.4} \rightarrow (4,3)_{+1}$



In updating (3,2) not using any knowledge about current value of (3,3)

Adaptive Dynamic Programming

Utilities of neighboring states are mutually constrained, Bellman equation:

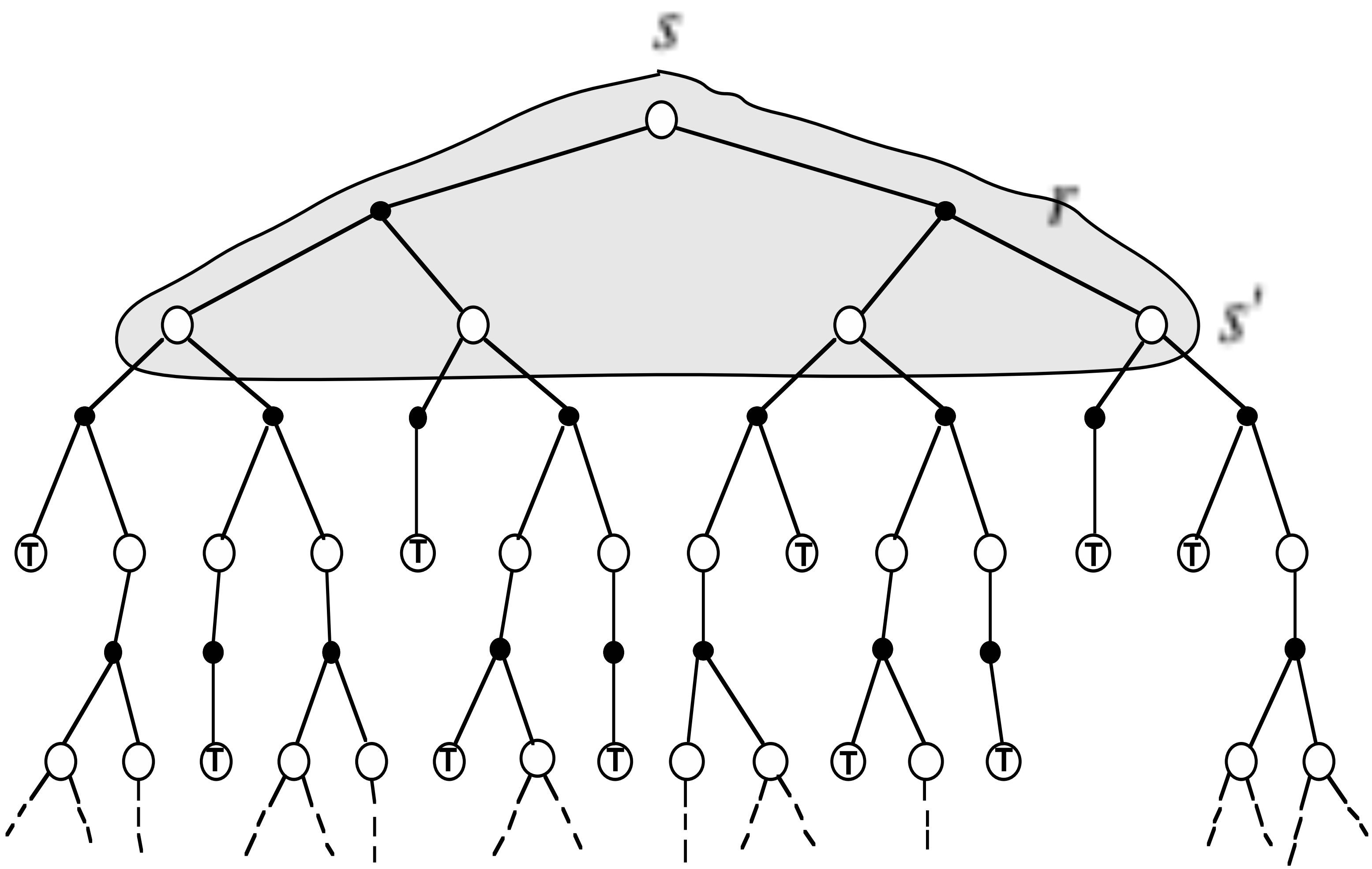
$$U(s) = R(s) + \gamma\, \Sigma_{s'} P(s,a,s')\, U(s')$$

Estimate P(s,a,s') from the frequency with which s' is reached when executing a in s.

Can use value iteration: initialize utilities based on the rewards and update all values based on the above equation.

Sometime intractable given a big state space.

Adaptive/Stochastic Dynamic Programming



TD: Temporal Difference Learning

- One of the first RL algorithms
- Learn the value of a *fixed* policy (no optimization; just prediction)
- Approximate the constraint equations without solving them for all states.

$$U^{\pi}(s) = R(s) + \gamma \sum_{s'} P(s, \pi(s), s') U^{\pi}(s')$$

Problem: We don't know this.

- Modify U(s) whenever we see a transition from s to s' using the following rule:

$$U(s) = U(s) + \alpha\,(R(s) + \gamma\, U(s') - U(s))$$

Temporal Difference Learning cont.

- $U(s) = U(s) + \alpha\ (\underbrace{R(s) + \gamma\ U(s') - U(s)}_{\text{TD Error}})$
  - The modification moves U(s) closer to satisfying the original equation.
  - α: learning rate, can be a function α (N(s)) that decreases as N(s) increases [number of times visting state s].
- Rewrite to get

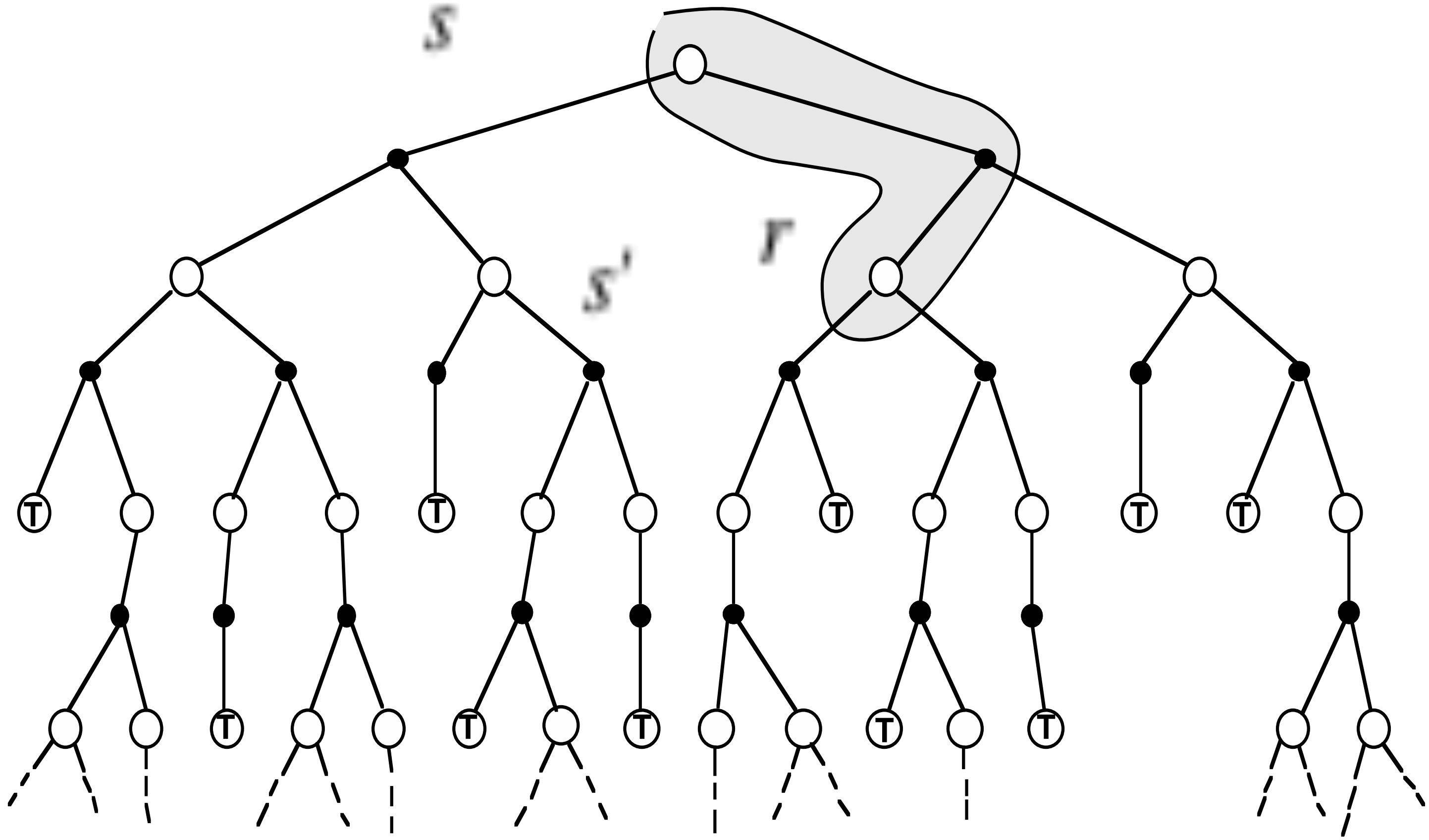
$$U(s)= (1-\alpha)\ U(s) + \alpha\ (\ R(s) + \gamma\ U(s')\ )$$

Temporal Difference (TD) Learning

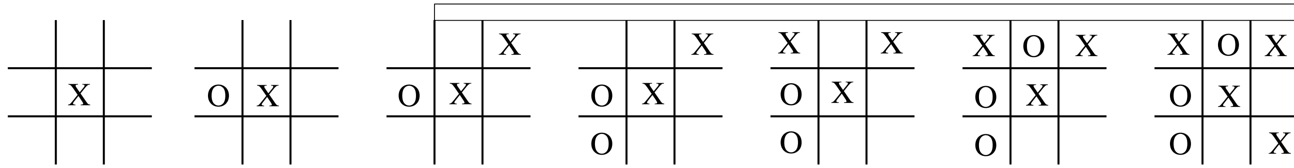
$$V(s) \leftarrow (1-\alpha)V(s) + \alpha\left[r + \gamma V(s')\right]$$

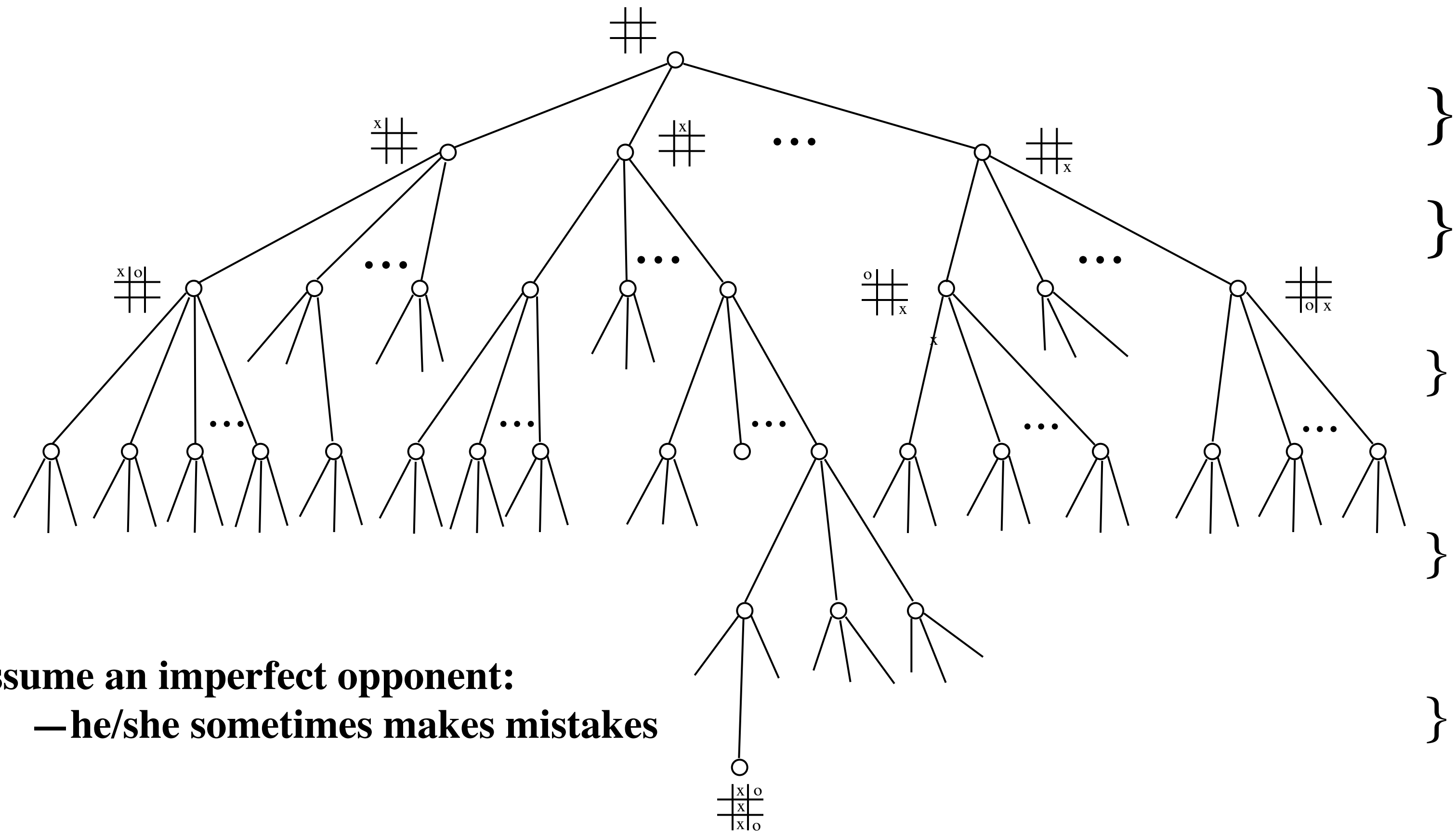
Sutton, 1988

After each action update the state



An Extended Example: Tic-Tac-Toe

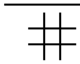
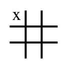
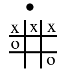
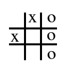
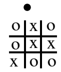




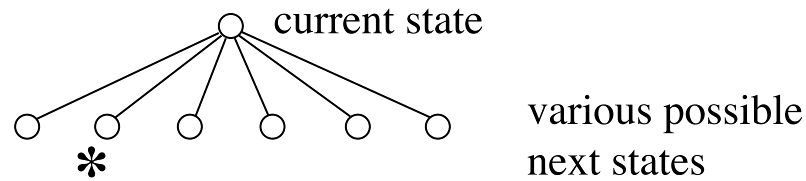
Assume an imperfect opponent:
—he/she sometimes makes mistakes

An RL Approach to Tic-Tac-Toe

1. Make a table with one entry per state:

State	$V(s)$ – estimated probability of winning	
	.5	?
	.5	?
⋮	⋮	
	1	win
⋮	⋮	
	0	loss
⋮	⋮	
	0	draw

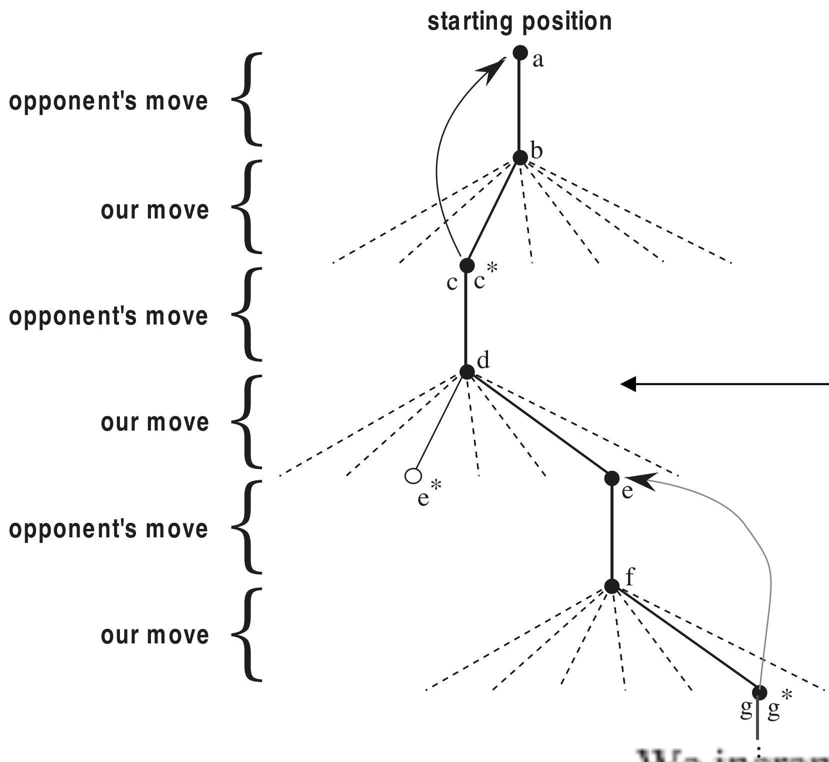
2. Now play lots of games.
To pick our moves,
look ahead one step:

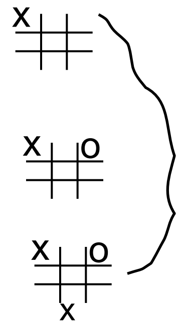


Just pick the next state with the highest estimated prob. of winning — the largest $V(s)$; a ***greedy*** move.

But 10% of the time pick a move at random; an ***exploratory move***.

RL Learning Rule for Tic-Tac-Toe





Take action O

Non-deterministic outcome based on opponent's move

"Exploratory" move

s – the state before our greedy move

s' – the state after our greedy move

We increment each $V(s)$ toward $V(s')$ – a ***backup***:

$$V(s) \leftarrow V(s) + \alpha\left[V(s') - V(s)\right]$$

a small positive fraction, e.g., $\alpha = .1$

the ***step - size parameter***

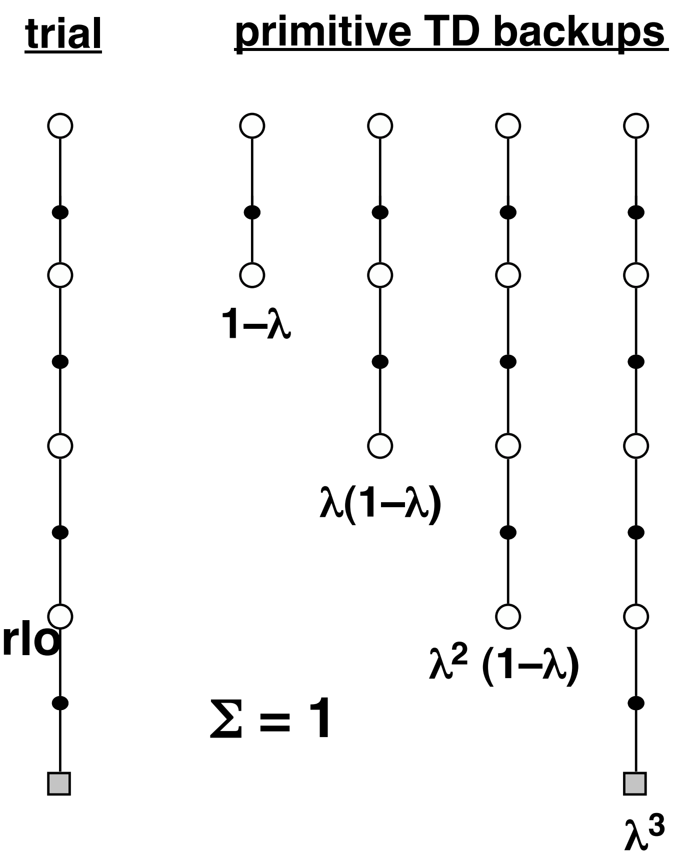
More Complex TD Backups

e.g. TD (λ)

Incrementally computes a weighted mixture of these backups as states are visited

$\lambda = 0 \longrightarrow$ Simple TD

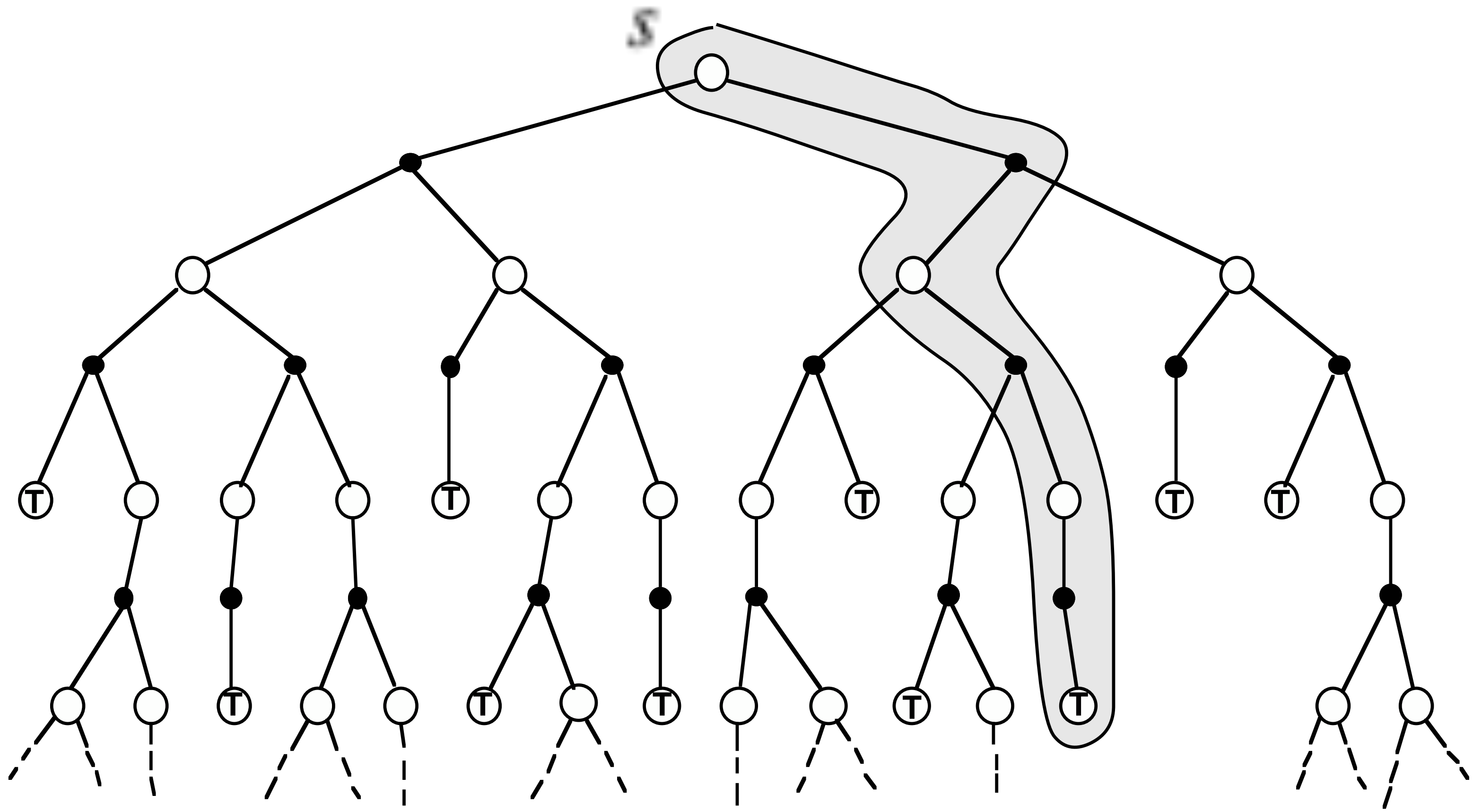
$\lambda = 1 \longrightarrow$ Simple Monte Carlo



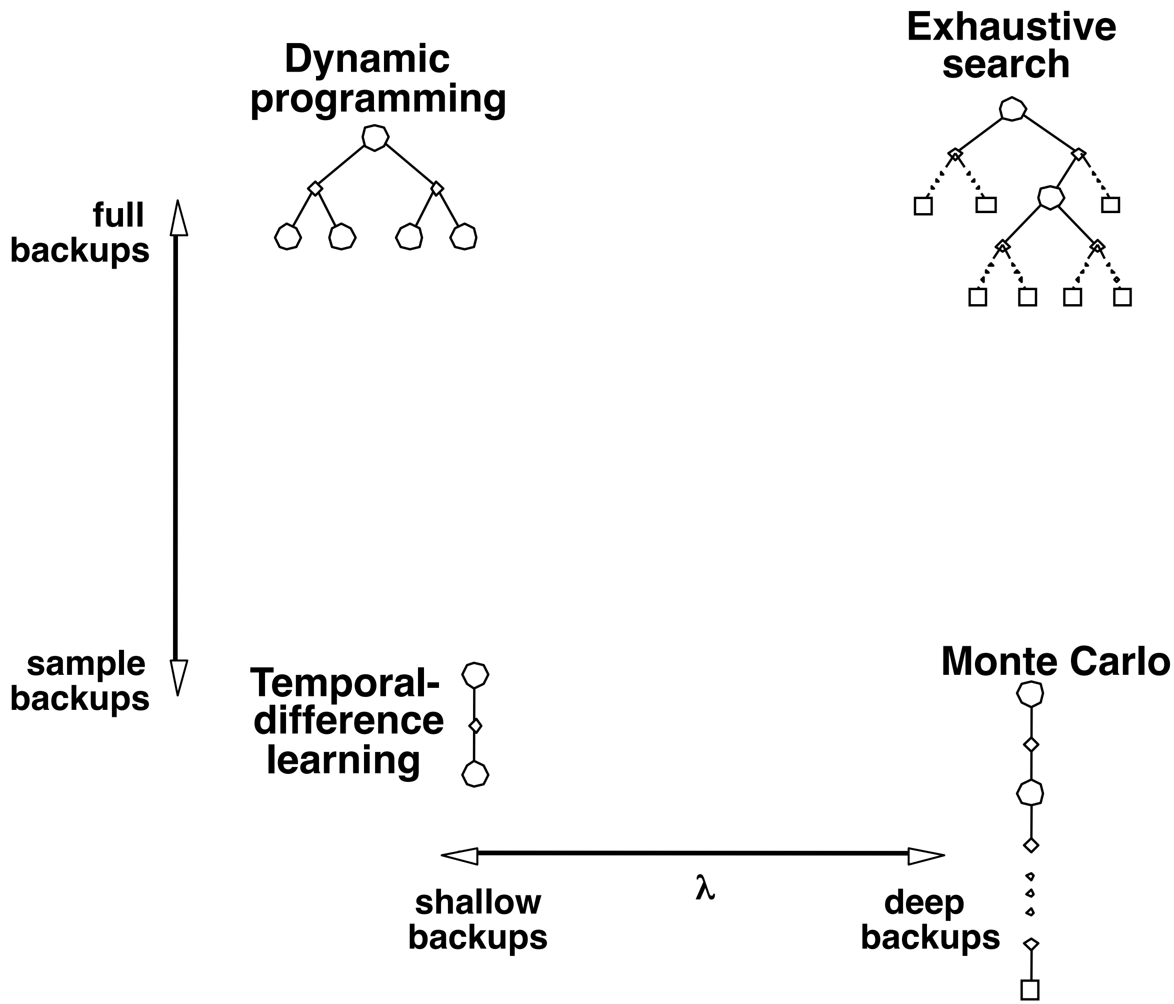
Blending the backups

Simple Monte Carlo

$$V(s) \leftarrow (1-\alpha)V(s) + \alpha\, REWARD(path)$$



Space of Backups



Backup from all terminal states

Limitation of Learning V^* Deterministic Case

Choose best action from any state *s using learned* V^*

$\pi^*(s)=\arg_a\max [r(s, a) + \gamma V^*(\delta(s, a))]$; deterministic case

A problem:

- This works well if agent knows $\delta: S \times A \rightarrow S$ and $r: S \times A \rightarrow \Re$
- But when it doesn't, it can't choose actions this way

How Much To do we Need to Know To Learn

Q Learning for Deterministic Case

Define new function very similar to V^*

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$

If agent learns Q, it can choose optimal action even without knowing r or δ!

$$\pi^*(s) = arg_a max[r(s,a) + \gamma V^*(\delta(s,a))]$$

$$\pi^*(s) = arg_a max Q(s,a)$$

Q is the evaluation function agent will learn

Training Rule to Learn Q for Deterministic Operators

Note Q and V^* closely related:

$$V^*(s) = \max_{a'} Q(s, a')$$

Which allows us to write Q recursively as

$$Q(s_t, a_t) = r(s_t, a_t) + \gamma V^*(\delta(s_t, a_t)))$$
$$= r(s_t, a_t) + \gamma \max_{a'} Q(s_{t+1}, a')$$

Let $\hat{Q}$ denote learner's current approximation to Q. Consider training rule

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

Where s' is the state resulting from applying action a in state s, and a' is the set of actions from s'

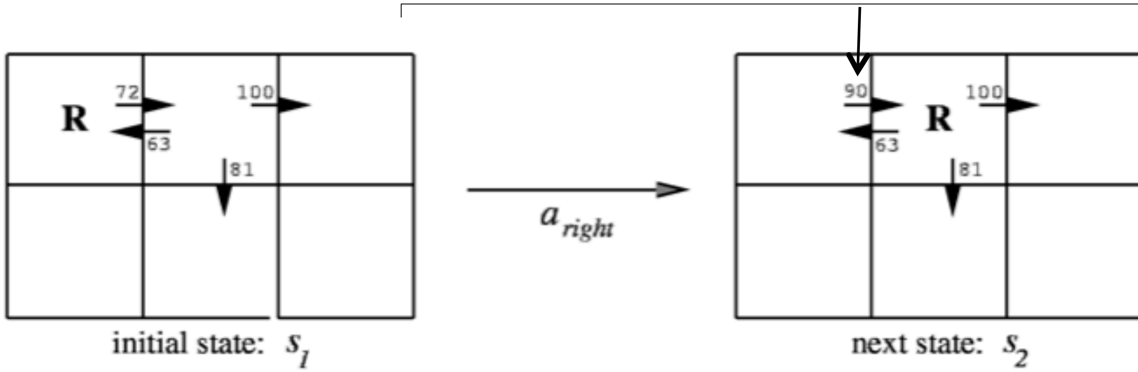
Q Learning for Deterministic Worlds

- For each s,a initialize table entry $\hat{Q}(s,a) \leftarrow 0$
- Observe current state s
- Do forever:
  - Select an action a and execute it
  - Receive immediate reward r
  - Observe the new state s'
  - Update the table entry for $\hat{Q}(s,a)$ as follows:

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

  - $s \leftarrow s'$

$\hat{Q}$ Updating



$$
\begin{aligned}
\hat{Q}(s_1, a_{right}) &\leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a') \\
&\leftarrow 0 + 0.9 \; \max\{63, 81, 100\} \\
&\leftarrow 90
\end{aligned}
$$

notice if rewards non-negative, then

$$(\forall s, a, n) \;\; \hat{Q}_{n+1}(s, a) \geq \hat{Q}_n(s, a)$$

and

$$(\forall s, a, n) \;\; 0 \leq \hat{Q}_n(s, a) \leq Q(s, a)$$

Nondeterministic Q learning Case

What if reward and next state are non-deterministic?

We redefine V,Q by taking expected values

$$V^{\pi}(s) \equiv E[r_t + \gamma r_{t+1} + \gamma r_{t+2} + \ldots]$$

$$\equiv E\left[\sum_{i=0}^{\infty} \gamma^i r_{t+i}\right]$$

$$Q(s,a) \equiv E[r(s, a) + \gamma V^*(\delta(s,a))]$$

Nondeterministic Case, cont'd

Q learning generalizes to non-deterministic worlds
Alter training rule to

$$\hat{Q}_n(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n[r + \max_{a'} \hat{Q}_{n-1}(s',a')]$$

Where $$\alpha_n = \frac{1}{1 + visits_n(s,a)}$$

Can still prove convergence of $\hat{Q}$ to Q
[Watkins and Dayan, 1992]

Q-learning cont.

- Is it better to learn a model and a utility function, or to learn an action-value function with no model?
- This is a fundamental question in AI where much of the research is based on a knowledge-based approach.
- Some researchers claim that the availability of model free methods such as Q-learning means that the KB approach is unnecessary (or too complex).

What actions to choose?

- Problem: choosing actions with the highest expected utility ignores their contribution to learning.
- *Tradeoff between immediate good and long-term good* (exploration vs. exploitation).
  - A random-walk agent learns faster but never uses that knowledge.
  - A greedy agent learns very slowly and acts based on current, inaccurate knowledge.

What's the best exploration policy?

- Give some weight to actions that were not tried very often in a given state, but counter that by knowledge that utility may be low.
  - Key idea is that in early stages of learning, estimations can be unrealistic low

- Similar to simulated annealing in that in the early phase of search more willing to explore

Practical issues - large State Set

- Too many states: Can define Q as a weighted sum of state features (factored state), or a neural net. Adjust the previous equations to update weights rather than updating Q.
  - Can have different neural networks for each action
  - This approach used very successfully in TD-Gammon (neural network).
- Continuous state-space: Can discretize it. Pole-balancing example (1968).

Reinforcement Learning Differs From Supervised Learning

- no presentation of input/output pairs
- agent chooses actions, receives reinforcement
- worlds are usually non-deterministic
- on-line performance is important
- system must explore the space of actions

End of Course

GOOD LUCK!!