Lecture 19: Uncertainty 4

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CMPSCI 683

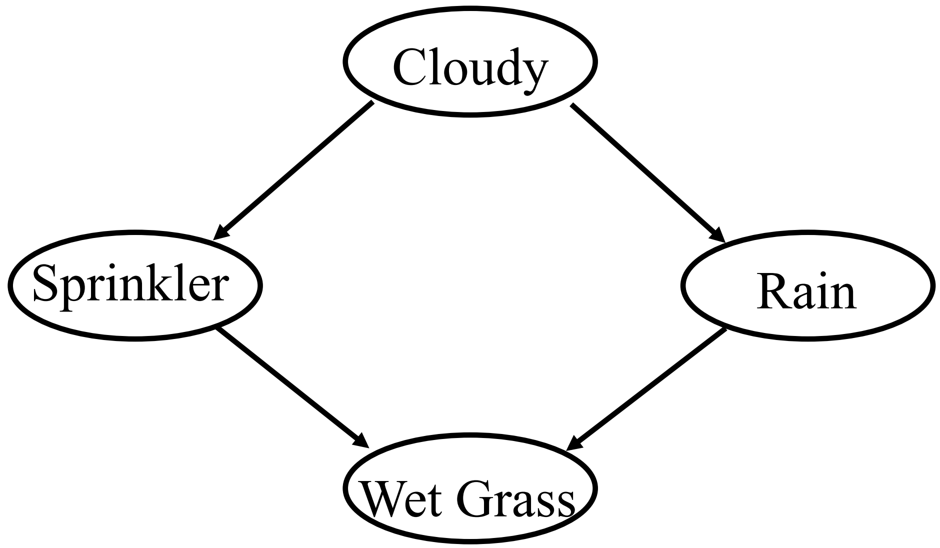
Fall 2010

Today's Lecture

- Inference in Multiply Connected BNs
  - **Clustering** methods transform the network into a probabilistically equivalent polytree.
    - Also called Join tree algorithms
  - **Conditioning** methods instantiate certain variables and evaluate a polytree for each possible instantiation.
  - **Stochastic simulation** approximate the beliefs by generating a large number of concrete models that are consistent with the evidence and CPTs.

Example of Multiply Connected BN

P(C=T)=.5



C	P(S=T)
T	.10
F	.50

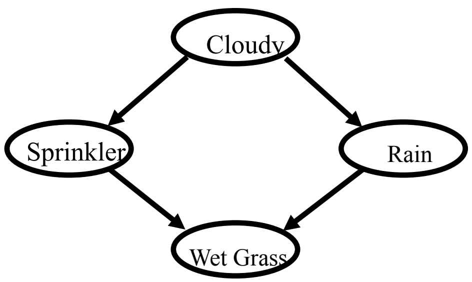
C	P(R=T)
T	.80
F	.20

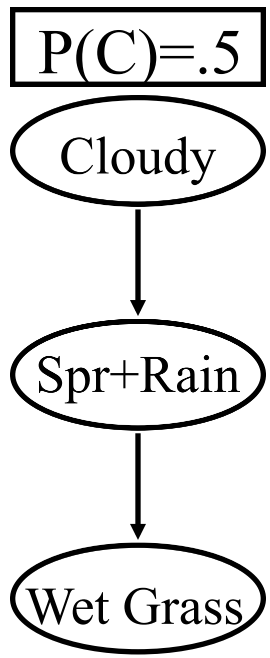
S	R	P(W=T)
T	T	.99
T	F	.90
F	T	.90
F	F	.00

Clustering Methods

- *Creating meganodes until the network becomes a polytree.*
- Most effective approach for exact evaluation of multiply connected BNs.
- The tricky part is choosing the right meganodes.
- Q. What happens to the NP-hardness of the inference problem?

Clustering Example*





C	P(S+R) TT	TF	FT	FF
T	.08	.02	.72	.18
F	.10	.40	.10	.40

S+R	P(W)
T T	.99
T F	.90
F T	.90
F F	.00

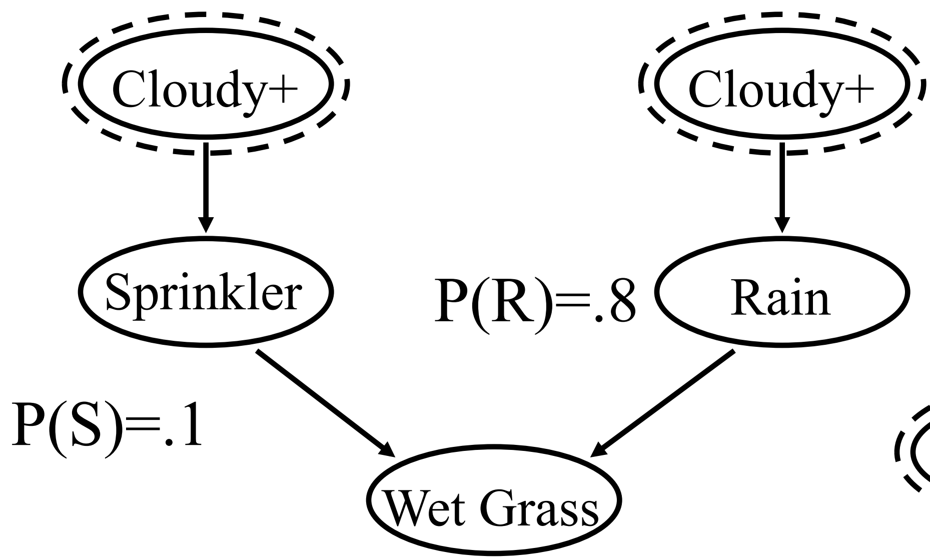
How do you still answer
P(Rain=True | Wet Grass=False) ?
How do you create meganode?
What are the disadvantages?

Cutset Conditioning Methods

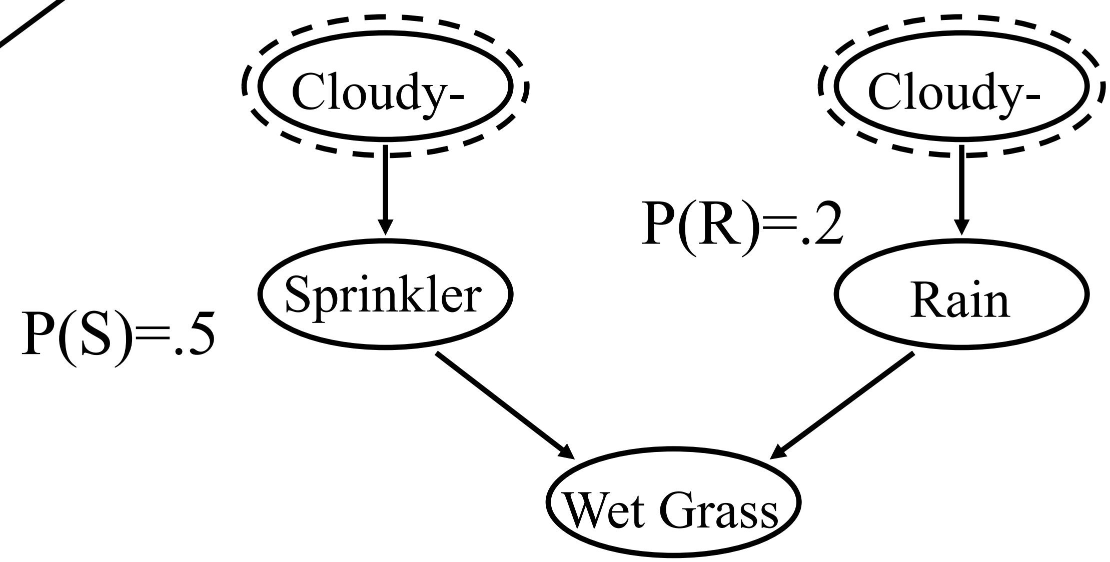
- *Once a variable is instantiated it can be duplicated and thus "break" a cycle.*
- A cutset is a set of variables whose instantiation makes the graph a polytree.
- Each polytree's likelihood is used as a weight when combining the results.

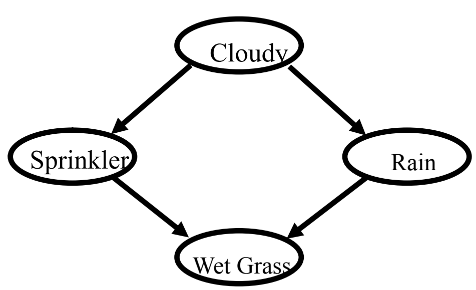
Networks Created by Instantiation

- Eliminate Cloudy from BN; Sum(%Cloudy+,%Cloud-)



C	P(S)	C	P(R)
T	.10	T	.80
F	.50	F	.20



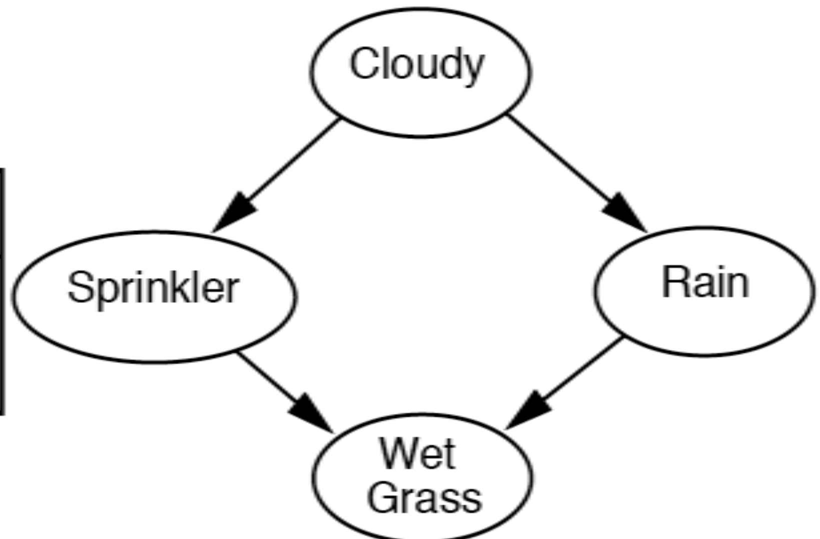


Stochastic Simulation -- Direct Sampling

- Assign each root node (without parents) a value based on prior probability.
- Assign all other nodes a NULL "value".
- Pick a node X with no value, but whose parents have values, and randomly assign a value to X
  - using P(X|Parents(X)) as the distribution. Repeat until there is no such X.
- After N trials, P(X|E) can be estimated by occurrences (X and E) / occurrences (E).
  - Approximate P(X,E)/P(E)
  - Does not focus on generating occurrences of E

Example P(WetGrass|Cloudy)

P(C)
.50



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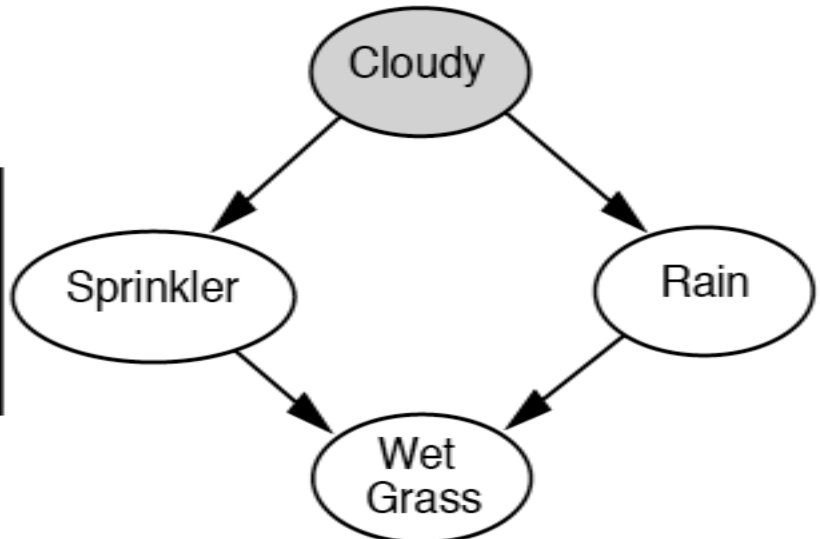
T	.80
F	.20

S	R	P(W|S,R)
T	T	.99
T	F	.90
F	T	.90
F	F	.01

Example cont.

P(C)
.50



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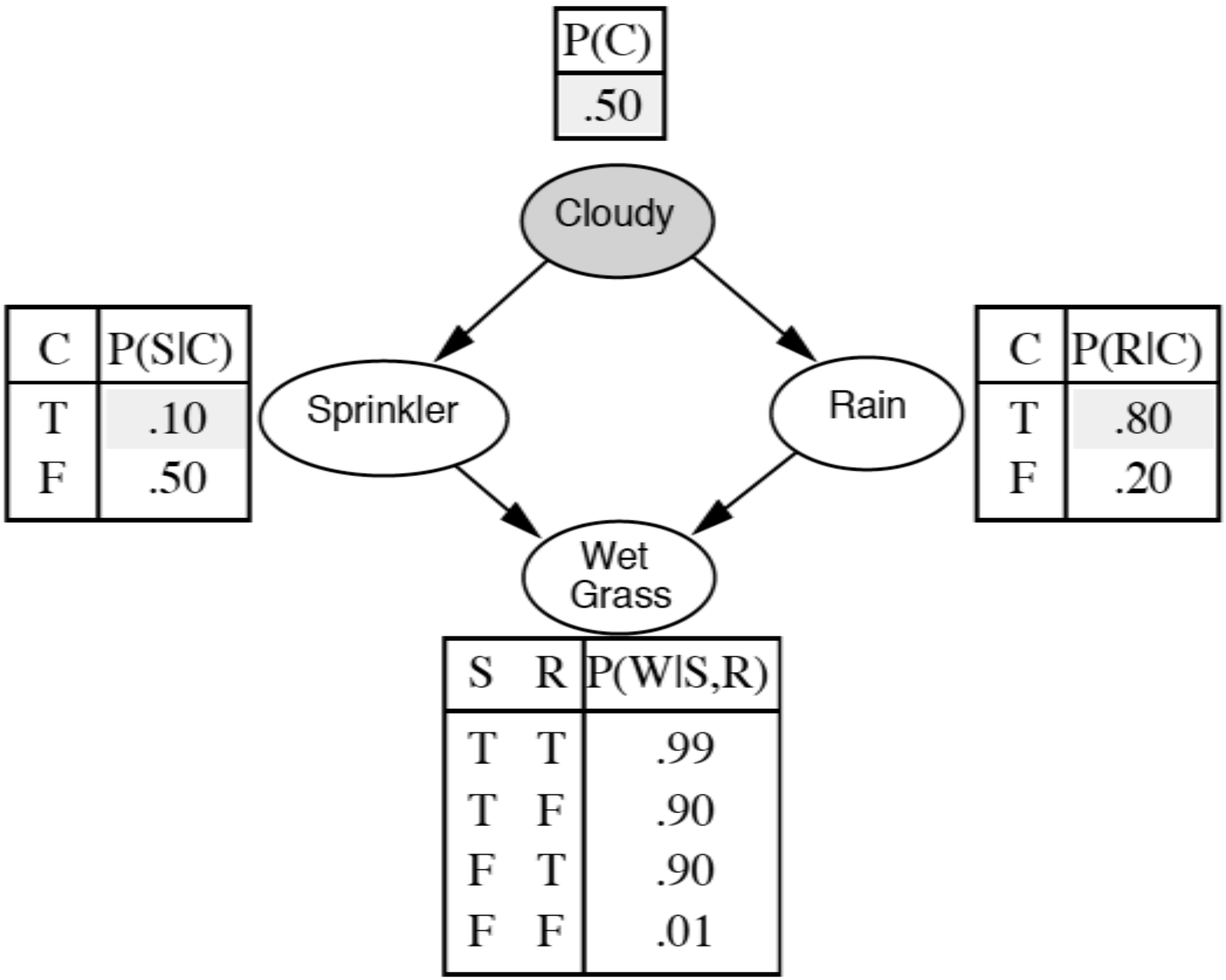
F	F	.01

Example cont.



Example cont.

P(C)
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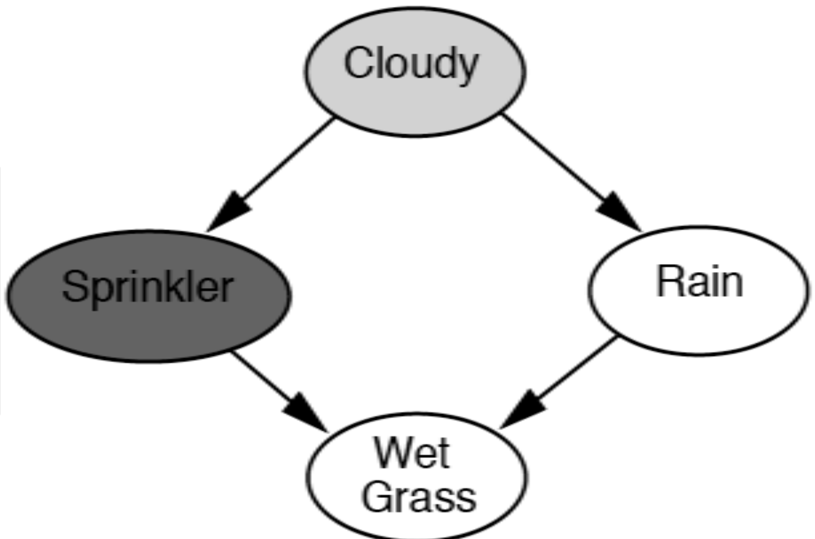

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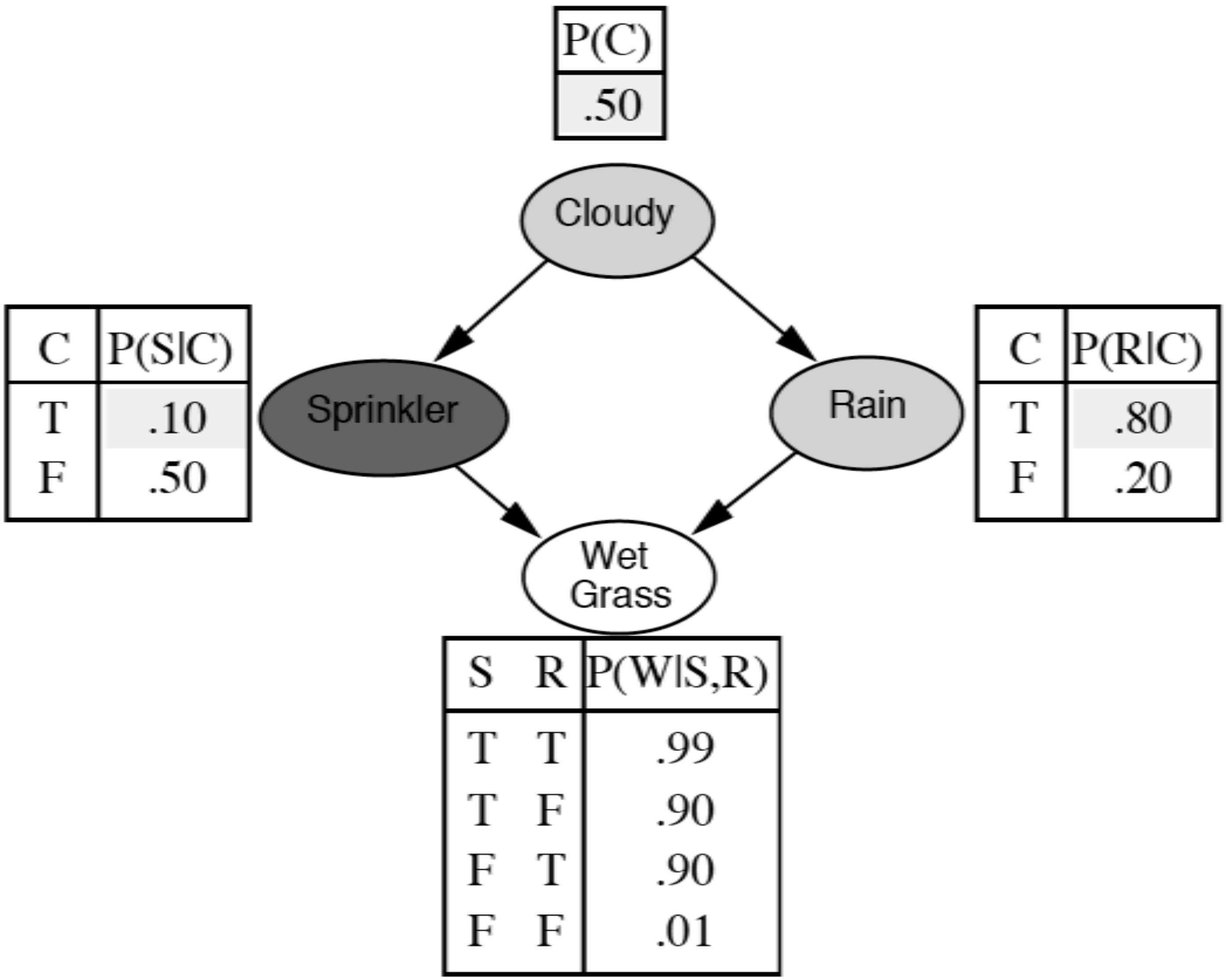
Example cont.



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Example cont.

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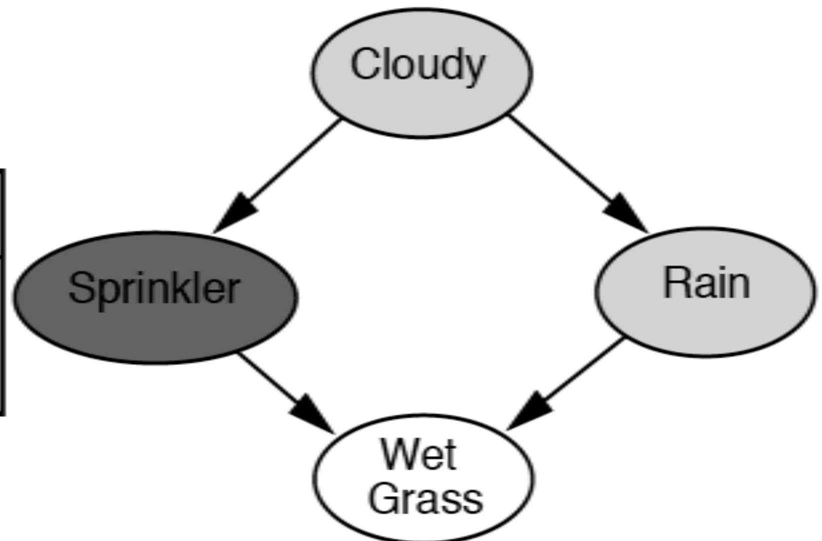

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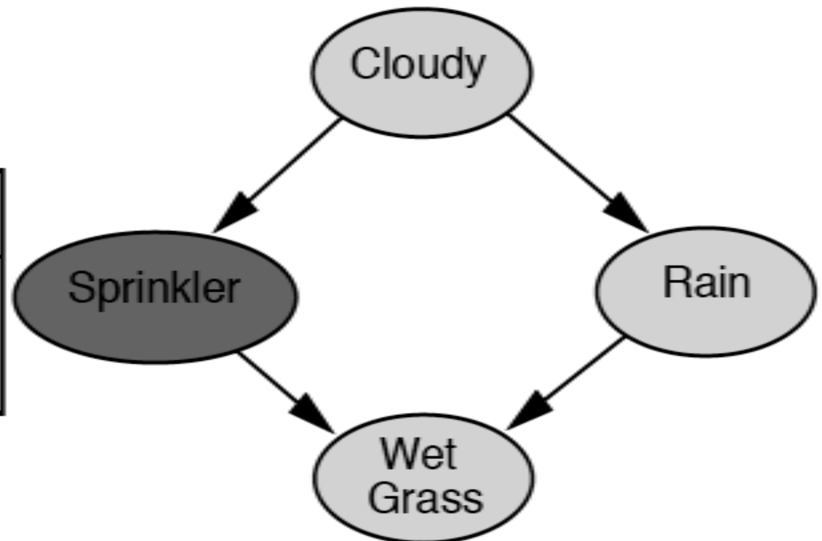
Example cont.



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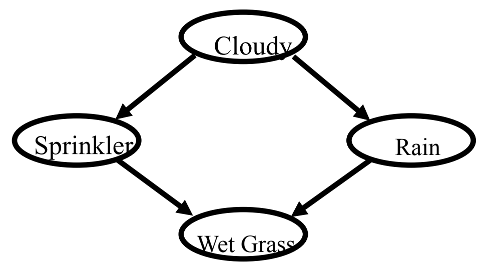
Stochastic Simulation cont.

- Problem with very unlikely events.
- Likelihood weighting can be used to fix problem.
- Likelihood weighting converges much faster than logic sampling and works well for very large networks.

Example of Likelihood Weighting

P(WetGrass | Rain)



- Choose a value for *Cloudy* with prior *P*(*Cloudy*) = 0.5. Assume we choose *cloudy* = *false*.
- Choose a value for *Sprinkler*. We see that *P(Sprinkler* | ¬ *Cloudy*) = 0.5, so we randomly choose a value given that distribution. Assume we choose *Sprinkler* =*True*.
- Look at *Rain*. This is an evidence variable that has been set to *True*, so we look at the table to see that *P*(*Rain* | ¬ *Cloudy*) = 0.2. This run therefore counts as 0.2 of a complete run.

Example of Likelihood Weighty cont'd

- Look at *WetGrass*. Choose randomly with $P(WetGrass \mid Sprinkler=T \wedge Rain=T) = 0.99$; assume we choose *WetGrass* = *True*.
- We now have completed a run with likelihood 0.2 that says *WetGrass* = *True* given *Rain* = *True.* The next run will result in a different likelihood, and (possibly) a different value for *WetGrass*. We continue until we have accumulated enough runs, and then add up the evidence for each value, **weighted by the likelihood score.**

Likelihood weighting usually converges much faster than logic sampling

Still takes a long time to reach accurate probabilities for unlikely events

Stochastic Simulation – Likelihood Weighting

Idea: fix evidence variables, sample only nonevidence variables, and weight each sample by the likelihood it accords the evidence

function LIKELIHOOD-WEIGHTING(X, $\mathbf{e}$, bn, N) **returns** an estimate of $P(X|\mathbf{e})$
  local variables: $\mathbf{W}$, a vector of weighted counts over X, initially zero

  for $j = 1$ to N **do**
    $\mathbf{x}$, $w \leftarrow$ WEIGHTED-SAMPLE(bn)
    $\mathbf{W}[x] \leftarrow \mathbf{W}[x] + w$ where x is the value of X in $\mathbf{x}$
  return NORMALIZE($\mathbf{W}[X]$)

---

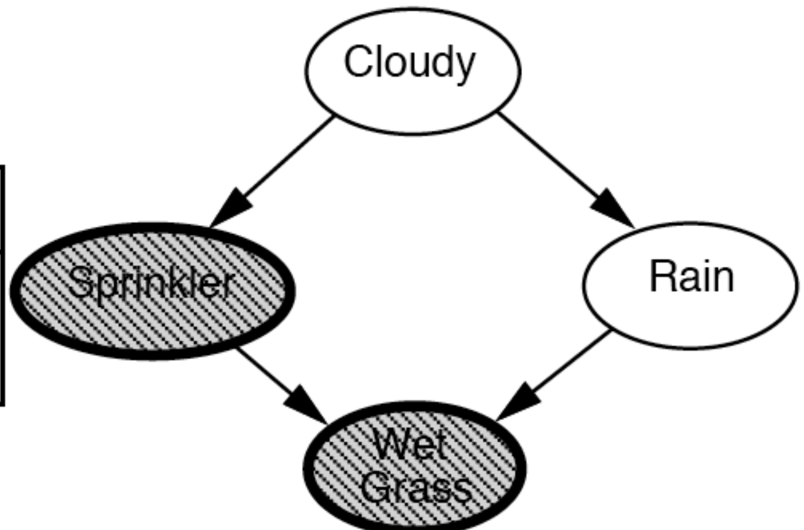
function WEIGHTED-SAMPLE(bn, $\mathbf{e}$) **returns** an event and a weight

  $\mathbf{x} \leftarrow$ an event with n elements; $w \leftarrow 1$
  for $i = 1$ **to** n **do** ; for all nodes in the network ordered by parents
    if X_i has a value x_i in $\mathbf{e}$; if you are at the node that you have evidence for
      then $w \leftarrow w \times P(X_i = x_i \mid Parents(X_i))$; adjust likelihood of this run based on the likelihood of evidence given parents
      else $x_i \leftarrow$ a random sample from $\mathbf{P}(X_i \mid Parents(X_i))$;otherwise randomly choose based on value of parents chosen in previous steps
  return $\mathbf{x}$, w

Likelihood weighting example

P(Rain| Sprinkler=T, WetGrass=T)

P(C)
.50



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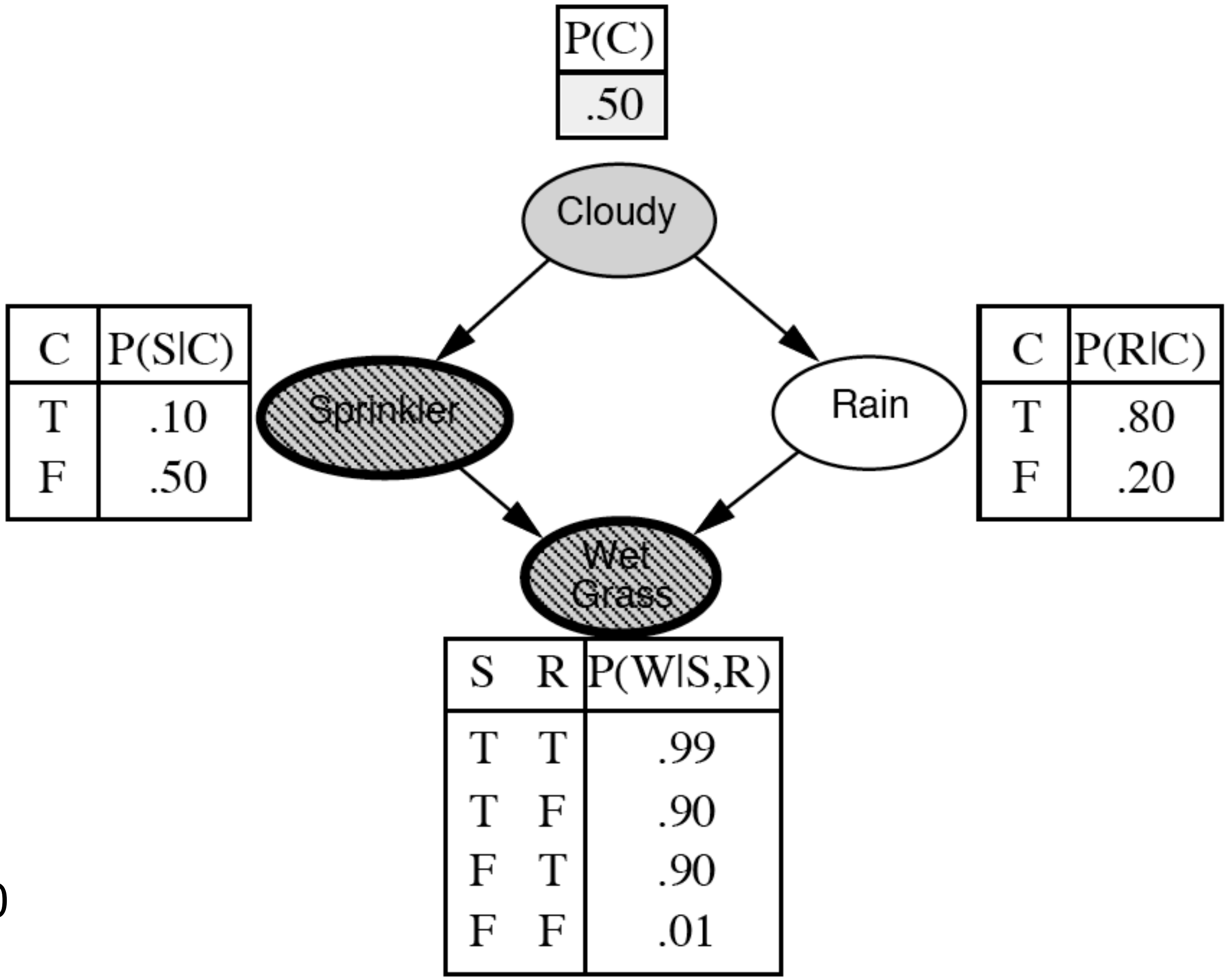
$w = 1.0$

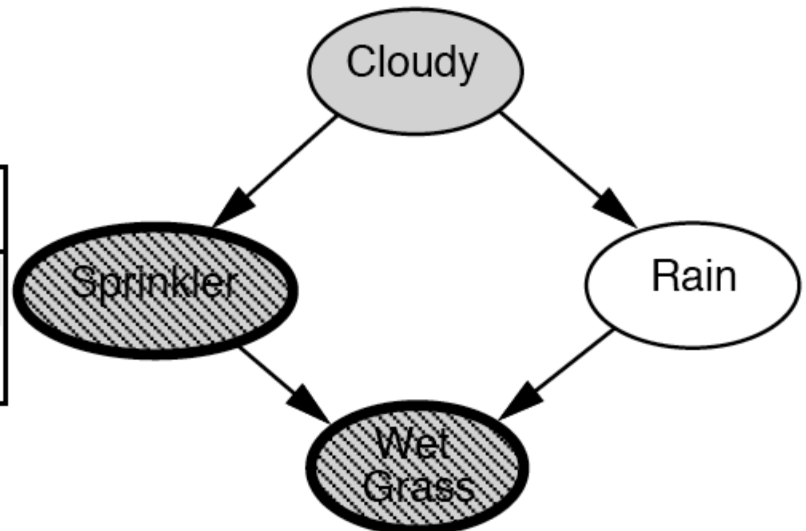
Example cont.



w = 1.0

Example cont.

P(C)
.50



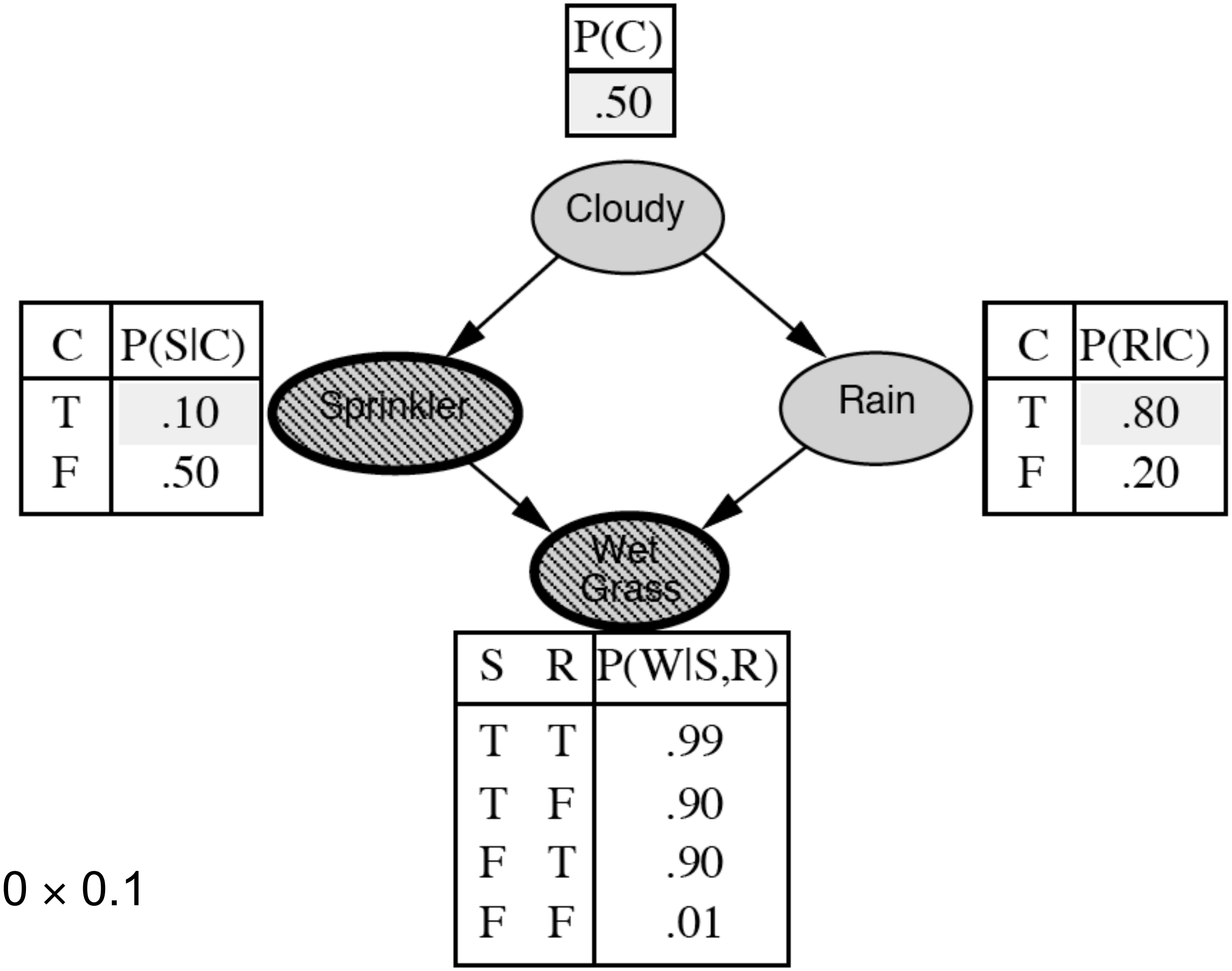
C	P(S|C)
T	.10
F	.50

C	P(R|C)
T	.80
F	.20

$w = 1.0 \times 0.1$

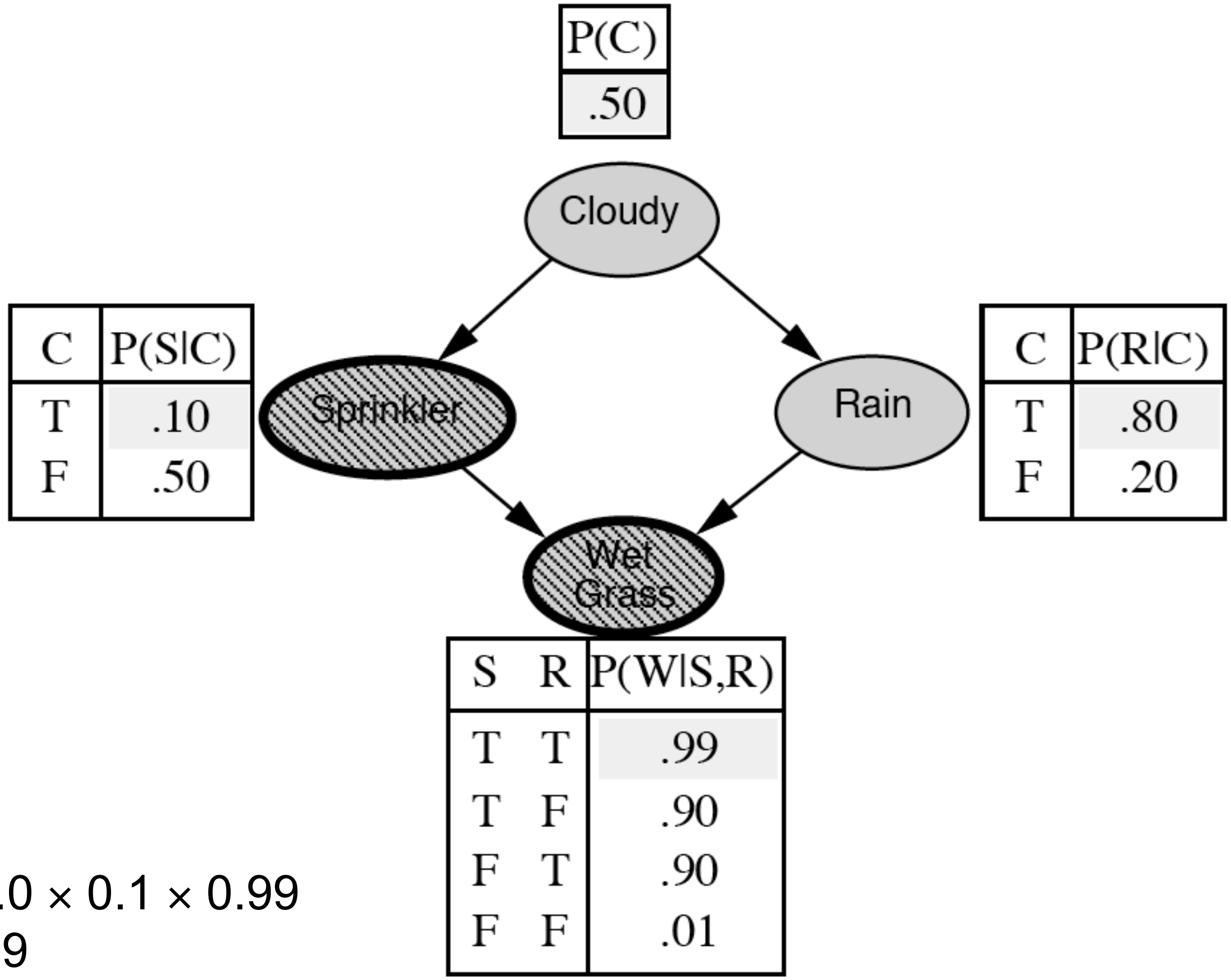
S	R	P(W|S,R)
T	T	.99
T	F	.90
F	T	.90
F	F	.01

Example cont.



$w = 1.0 \times 0.1$

Example cont.



P(C)
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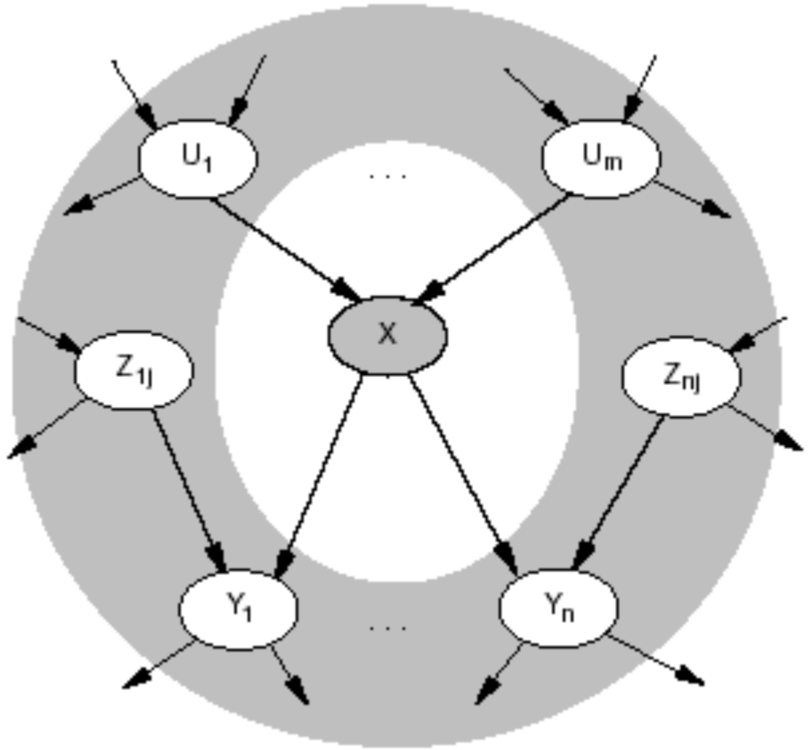
F	T	.90
F	F	.01

$W = 1.0 \times 0.1 \times 0.99$
$= 0.099$

Stochastic Simulation – Markov Chain Monte Carlo



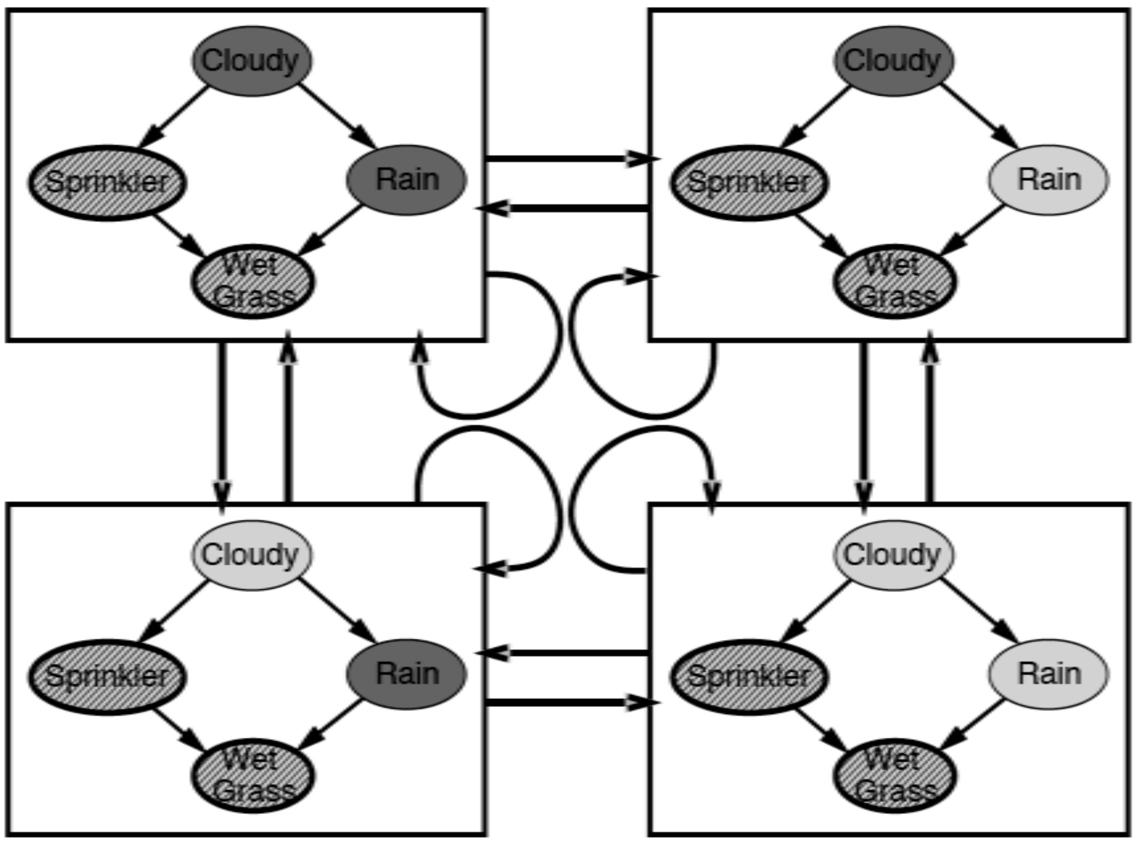
A node is conditionally independent of all other nodes in the network given its parents, children, and children's parents —that is, given its **Markov blanket**.

The MCMC algorithm

- MCMC generates each event by making a random change to the preceding event.
  - It is therefore helpful to think of the network being in a particular *current state* specifying a value for every variable.
- The next state is generated by randomly sampling a value for one of the non-evidence variables X_i, *conditioned on the current values of the variables in the Markov blanket of* X_i.
  - Don't need to look at any other variables
- MCMC therefore wanders randomly around the state space—the space of possible complete assignments—flipping one variable at a time but keeping the evidence variables fixed.

The Markov chain

With $Sprinkler = true, WetGrass = true$, there are four states:



Wander about for a while, average what you see

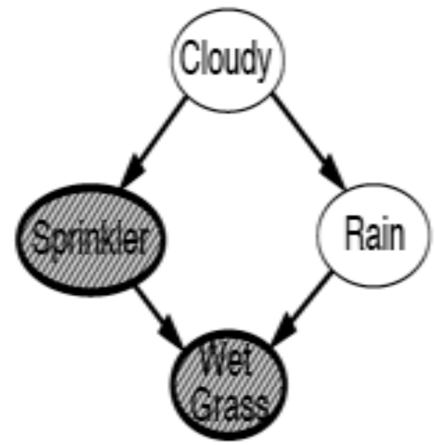
Markov blanket sampling

Markov blanket of *Cloudy* is

Sprinkler and *Rain*

Markov blanket of *Rain* is

Cloudy, *Sprinkler*, and *WetGrass*



Probability given the Markov blanket is calculated as follows:

$$P(x_i'|MB(X_i)) = P(x_i'|Parents(X_i))\prod_{Z_j \in Children(X_i)} P(z_j|Parents(Z_j))$$

MCMC example cont.

Estimate $\mathbf{P}(Rain|Sprinkler = true, WetGrass = true)$

Sample $Cloudy$ or $Rain$ given its Markov blanket, repeat.
Count number of times $Rain$ is true and false in the samples.

E.g., visit 100 states
31 have $Rain = true$, 69 have $Rain = false$

$$\hat{\mathbf{P}}(Rain|Sprinkler = true, WetGrass = true)$$
$$= \textsc{Normalize}(\langle 31, 69 \rangle) = \langle 0.31, 0.69 \rangle$$

Theorem: chain approaches stationary distribution:
long-run fraction of time spent in each state is exactly proportional to its posterior probability

Summary of a Belief Networks

- **Conditional independence** information is a vital and robust way to structure information about an uncertain domain.
- **Belief networks** are a natural way to represent conditional independence information.
  - The links between nodes represent the qualitative aspects of the domain, and the conditional probability tables represent the quantitative aspects.
- A belief network is a complete representation for the joint probability distribution for the domain, but is often exponentially smaller in size.

Summary of a Belief Networks, cont'd

- Inference in belief networks means computing the probability distribution of a set of query variables, given a set of evidence variables.

- Belief networks can reason causally, diagnostically, in mixed mode, or intercausally. No other uncertain reasoning mechanism can handle all these modes.

- The complexity of belief network inference depends on the network structure. In **polytrees** (singly connected networks), the computation time is linear in the size of the network.

Summary of a Belief Networks, cont'd

- There are various inference techniques for general belief networks, all of which have exponential complexity in the worst case.
  - In real domains, the local structure tends to make things more feasible, but care is needed to construct a tractable network with more than a hundred nodes.
- It is also possible to use approximation techniques, including **stochastic simulation,** to get an estimate of the true probabilities with less computation.

Next Lecture

- Introduction to Decision Theory
  - Making Single-Shot Decisions
  - Utility Theory