Lecture 12: MDP1

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Biased Random GSAT - WalkSat

function WALKSAT(*clauses*, *p*, *max_flips*) **returns** a satisfying model or *failure*
  inputs: *clauses*, a set of clauses in propositional logic
    p, the probability of choosing to do a "random walk" move, typically around 0.5
    max_flips, number of flips allowed before giving up

  model ← a random assignment of *true*/*false* to the symbols in *clauses*
  for i = 1 **to** *max_flips* **do**
    if *model* satisfies *clauses* **then return** *model*
    clause ← a randomly selected clause from *clauses* that is false in *model*
    with probability *p* flip the value in *model* of a randomly selected symbol from *clause*
    else flip whichever symbol in *clause* maximizes the number of satisfied clauses
  return *failure*

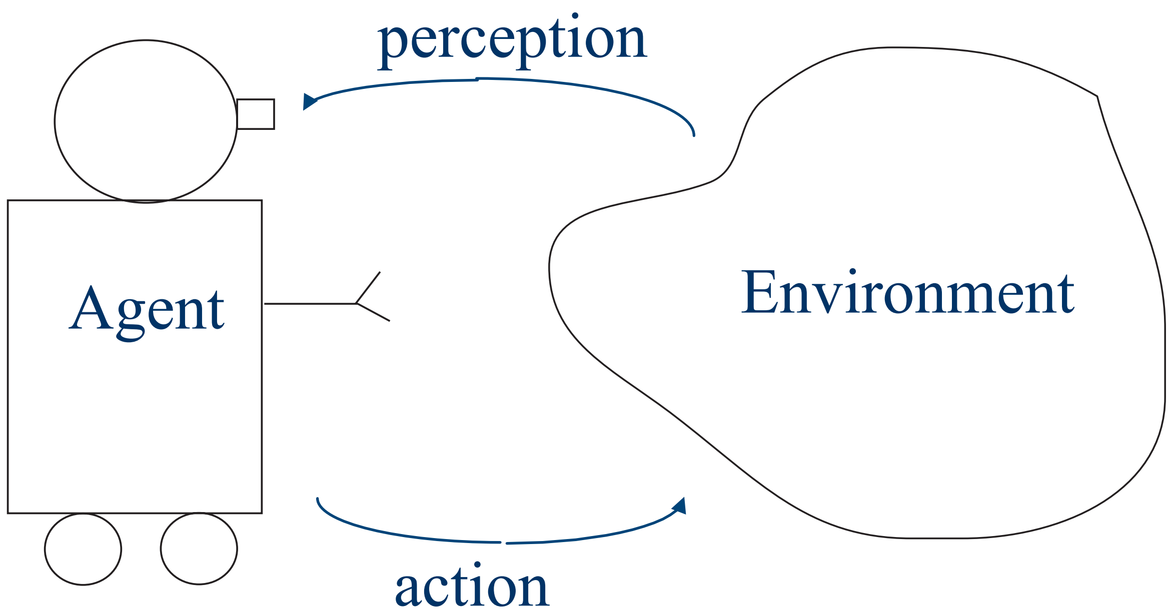
Notice no random restart

Figure 7.18 The WALKSAT algorithm for checking satisfiability by randomly flipping the values of variables. Many versions of the algorithm exist.

Today's lecture

- Search where there is Uncertainty in Operator Outcome --Sequential Decision Problems
    - Planning Under Uncertainty
  - Markov Decision Processes (MDP)

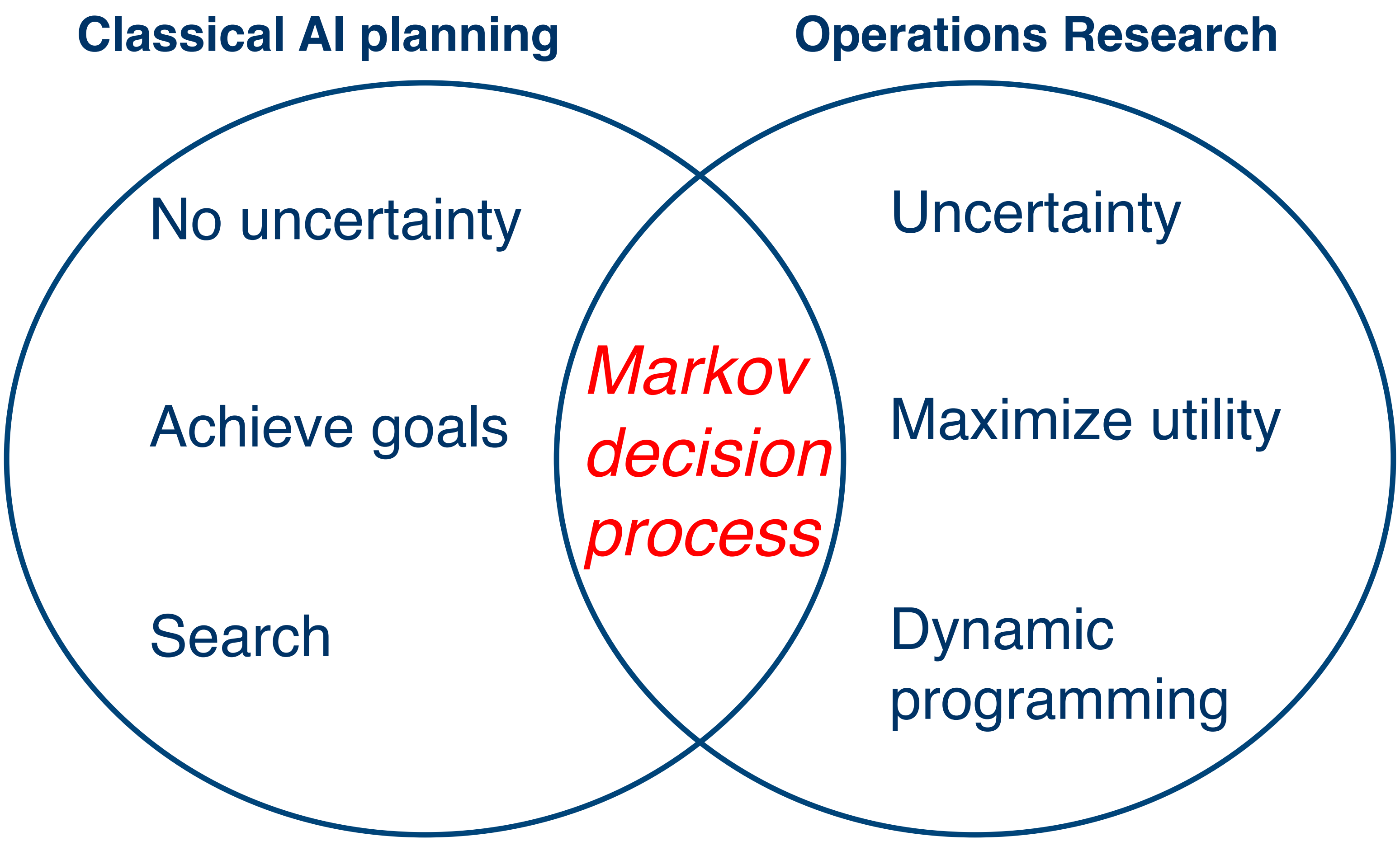
Planning under uncertainty



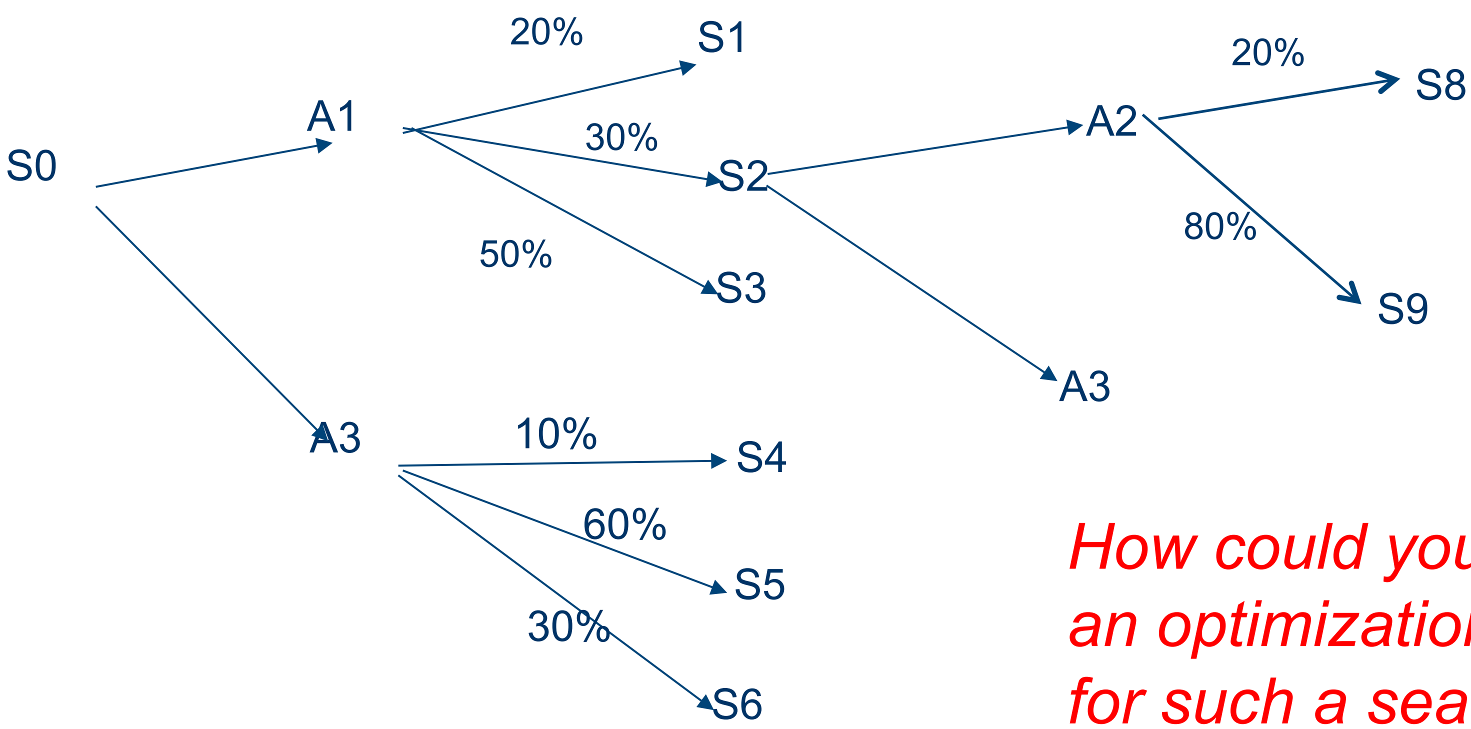
Utility depends on a sequence of decisions

Actions have unpredictable outcomes

Approaches to planning



Search with Uncertainty



How could you define an optimization criteria for such a search?

What is the output of the search?

Stochastic shortest-path problems

- Given a start state, the objective is to minimize the expected cost of reaching a goal state.
- S: a finite set of states
- $A(i)$, $i \in S$: a finite set of actions available in state i
- $P_{ij}(a)$: probability of reaching state j after action a in state i
- $C_i(a)$: expected cost of taking action a in state i

Markov decision process

- A model of sequential decision-making developed in operations research in the 1950's.
- Allows reasoning about actions with uncertain outcomes.
- MDPs have been adopted by the AI community as a framework for:
  - Decision-theoretic planning (e.g., [Dean et al., 1995])
  - Reinforcement learning (e.g., [Barto et al., 1995])

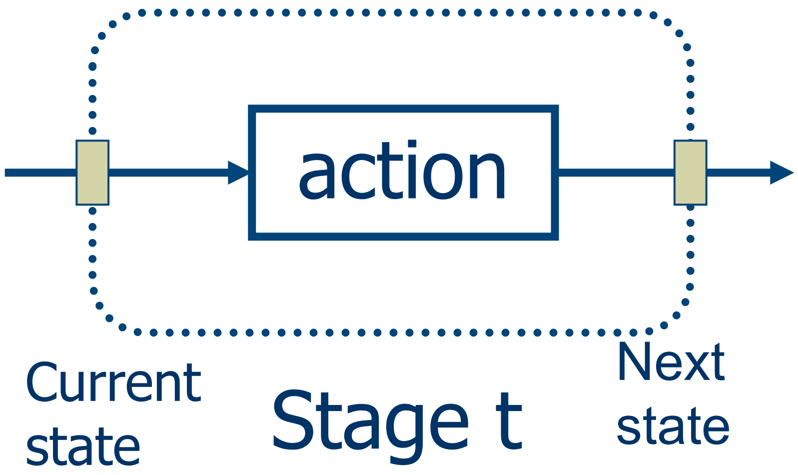
Markov Decision Processes (MDP)

- S - finite set of domain states
- A - finite set of actions
- $\mathrm{P}(s' \mid s, a)$ - state transition function
- $R(s)$, $R(s, a)$, or $R(s, a, s')$ - reward function
  - *Could be negative to reflect cost*
- S_0 - initial state
- The Markov assumption:

$$\mathrm{P}(s_t \mid s_{t-1}, s_{t-2}, \ldots, s_1, a) = \mathrm{P}(s_t \mid s_{t-1}, a)$$

The MDP Framework (cont)

- Agent *fully observes its current state*
- Markovian property: the state contains enough information to pick the optimal action
- Objective: maximize the expected reward of the start state
- Policy: π : S --> A; how to find optimal policy



Policy vs. Plan

A Finite MDP with Loops

Recycling Robot

- At each step, robot has to decide whether it should
  - (1) actively search for a can.
  - (2) wait for someone to bring it a can.
  - (3) go to home base and recharge.
- Searching is better but runs down the battery; if runs out of power while searching, has to be rescued (which is bad and represented as a penalty).
- Decisions made on basis of current energy level: `high`, `low`.
- Reward = number of cans collected

Recycling Robot MDP

$S = \{\texttt{high}, \texttt{low}\}$

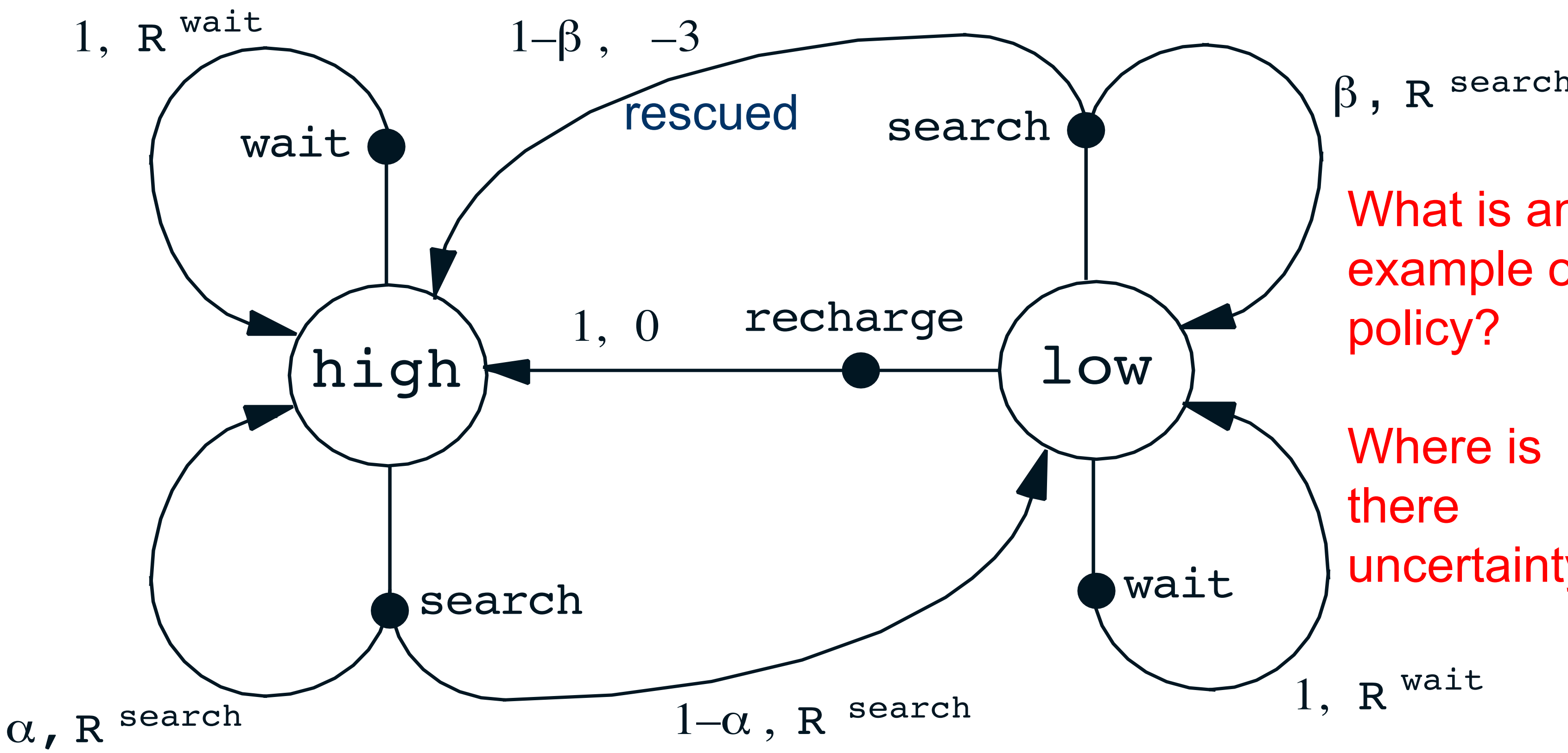
$A(\texttt{high}) = \{\texttt{search}, \texttt{wait}\}$

$A(\texttt{low}) = \{\texttt{search}, \texttt{wait}, \texttt{recharge}\}$

$R^{\texttt{search}}$ = expected no. of cans while searching

$R^{\texttt{wait}}$ = expected no. of cans while waiting

$R^{\texttt{search}} > R^{\texttt{wait}}$

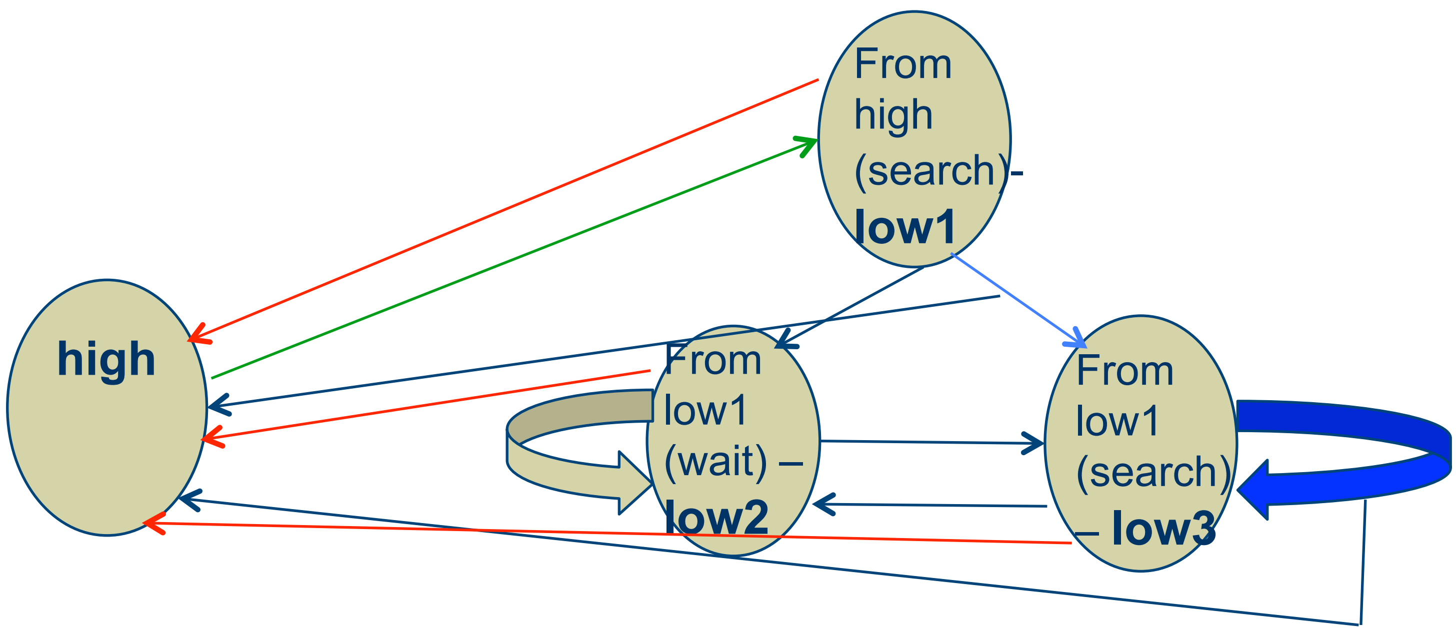


What is an example of a policy?

Where is there uncertainty?

Breaking the Markov Assumption to get a Better Policy

- Concerned about path to Low State (whether you came as a result of a search from a high state or a search or wait action from a low state (high, low1, low2, low3)
  - can more accurately reflect likelihood of rescue
  - develop policy that does one search in low state



Goals and Rewards

- Is a scalar reward signal an adequate notion of a goal?—maybe not, but it is surprisingly flexible.
- A goal should specify **what** we want to achieve, not **how** we want to achieve it.
  - It is not the path to a specific state but reaching a specific state – fits with Markov Assumption
- A goal must be outside the agent's direct control—thus outside the agent.
- The agent must be able to measure success:
  - Explicitly in terms of a reward;
  - frequently during its lifespan.

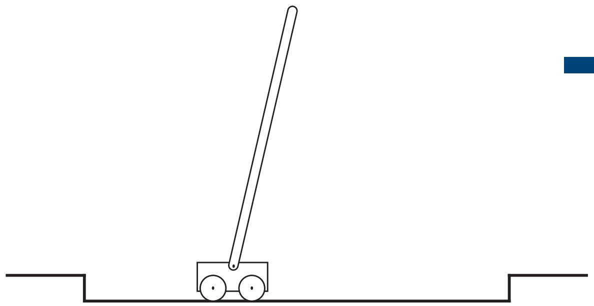
Performance criteria

- Specify how to combine rewards over multiple time steps or histories.
- Finite horizon problems involve a fixed number of steps.
- The best action in each state may depend on the number of steps left, hence it is **non-stationary**.
  - Finite horizon non-stationary problems can be solved by adding the number of steps left to the state – adds more states
- Infinite horizon policies depend only on the current state, hence the optimal policy is **stationary**.

Performance criteria cont.

- The assumption the agent's preferences between state sequences is stationary: $[s_0,s_1,s_2,\ldots] > [s_0,s_1',s_2',\ldots]$ iff $[s_1,s_2,\ldots] > [s_1',s_2',\ldots]$
  - *how you got to a state does not affect the best policy from that state*
- This leads to just two ways to define utilities of histories:
  - Additive rewards: utility of a history is $U([s_0,a_1,s_1,a_2,s_2,\ldots]) = R(s_0) + R(s_1) + R(s_2) + \ldots$
  - Discounted rewards: utility of a history is $U([s_0,a_1,s_1,a_2,s_2,\ldots]) = R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) \ldots$
- With a **proper policy** (guaranteed to reach a terminal state) no discounting is needed.
- An alternative to discounting in infinite-horizon problems is to optimize the average reward per time step.

An Example



Avoid **failure:** the pole falling beyond a critical angle or the cart hitting end of track.

As an **episodic task** where episode ends upon failure:

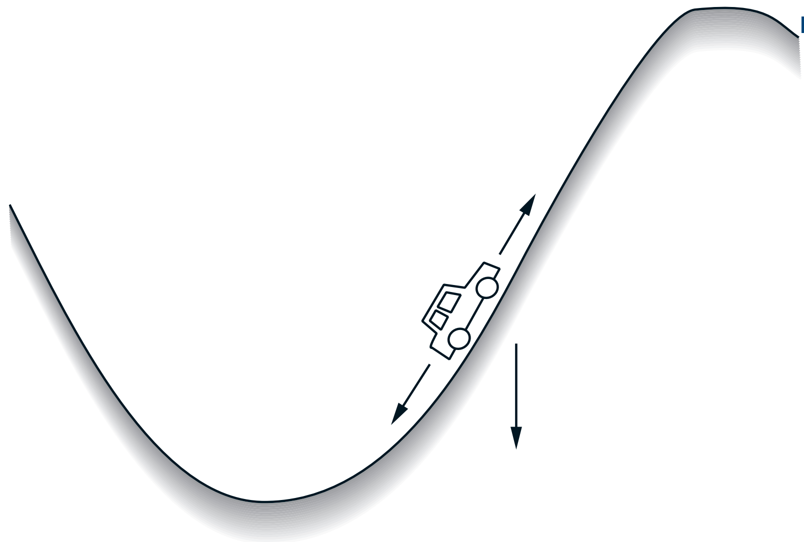
$$\text{reward} = +1 \text{ for each step before failure}$$
$$\Rightarrow \text{return} = \text{number of steps before failure}$$

As a **continuing task** with discounted return:

$$\text{reward} = -1 \text{ upon failure; } 0 \text{ otherwise}$$
$$\Rightarrow \text{return} = -\gamma^{k}, \text{ for } k \text{ steps before failure}$$

In either case, return is maximized by avoiding failure for as long as possible.

Another Example



Get to the top of the hill as quickly as possible.

reward = –1 for each step where **not** at top of hill

$\Rightarrow$ return = – number of steps before reaching top of hill

Return is maximized by minimizing number of steps to reach the top of the hill.

Policies and utilities of states

- A policy π is a mapping from states to actions.
- An optimal policy π* maximizes the expected reward:

$$\pi^* = \arg\max_{\pi} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid \pi \right]$$

- The utility of a state

$$U^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid \pi, s_0 = s \right]$$

A simple grid environment

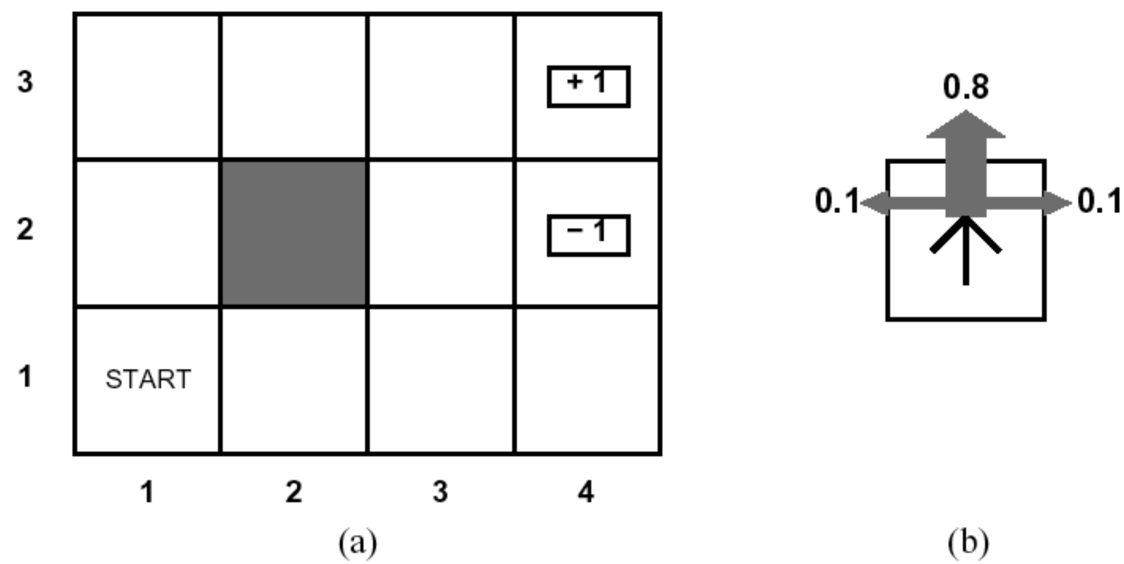


Figure 17.1 (a) A simple 4×3 environment that presents the agent with a sequential decision problem. (b) Illustration of the transition model of the environment: the "intended" outcome occurs with probability 0.8, but with probability 0.2 the agent moves at right angles to the intended direction.

Example: An Optimal Policy

A policy is a choice of what action to choose at each state

An Optimal Policy is a policy where you are always choosing the action that maximizes the “return”/”utility” of the current state

→	→	→	+1
↑		↑	−1
↑	←	←	←

.812	.868	.912	+1
.762		.660	−1
.705	.655	.611	.388

Actions succeed with probability 0.8 and *move at right angles with probability 0.1* (remain in the same position when there is a wall). Actions incur a small cost (0.04).

- What happens when cost increases?
- Why move from .611 to .655 instead of .660?

Policies for different $R(s)$

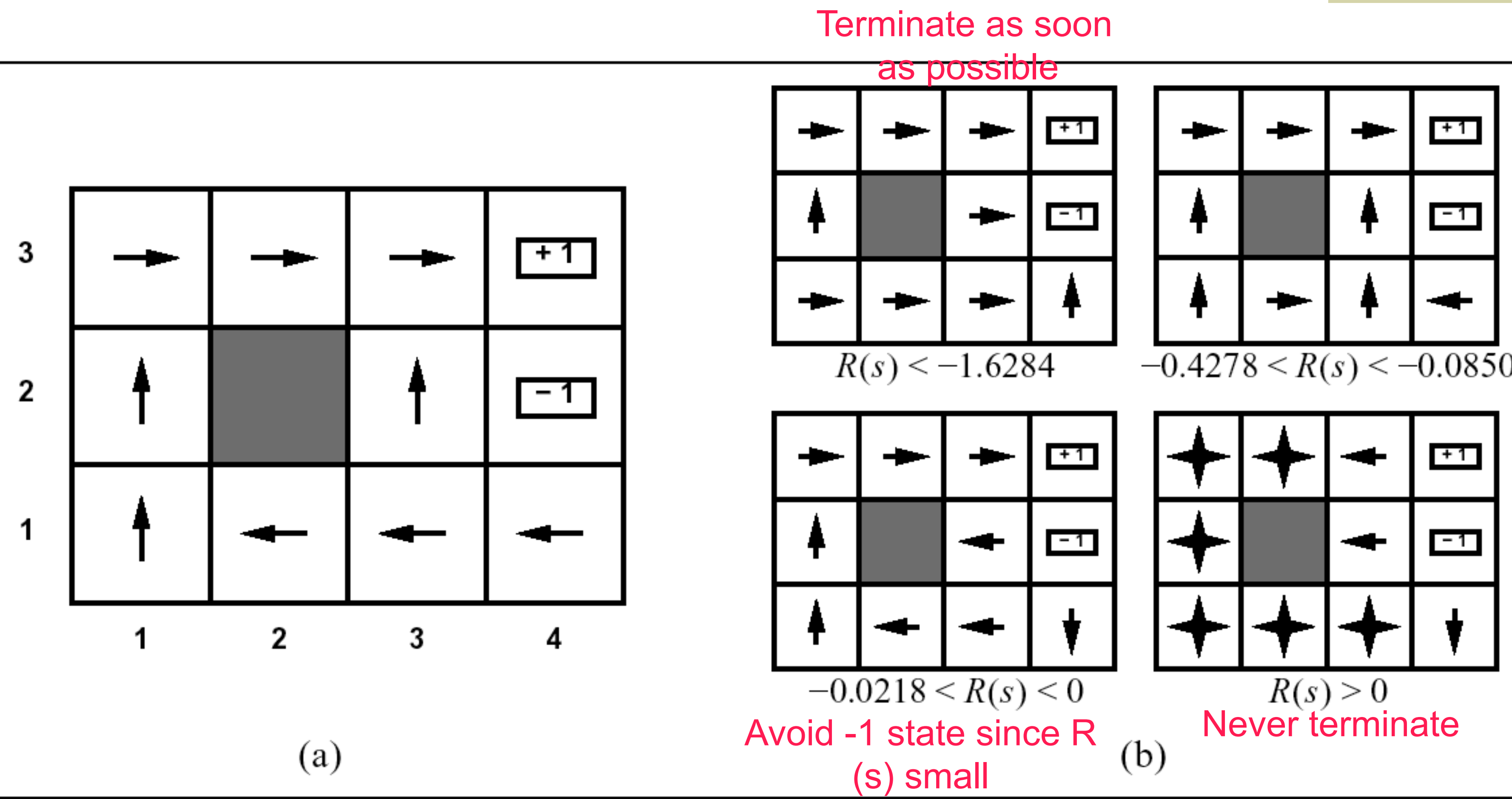


Figure 17.2 (a) An optimal policy for the stochastic environment with $R(s) = -0.04$ in the nonterminal states. (b) Optimal policies for four different ranges of $R(s)$.

Next Lecture

- Continuations with MDP
  - Value and policy iteration
- Search where is Uncertainty in

Operator Outcome and Initial State

Partial Orderded MDP (POMDP)

- Hidden Markov Processes